

Statistical Analysis of Timing Errors

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Human rhythmic activities are variable. Cycle-to-cycle fluctuations form the behavioral observable. Traditional analysis focuses on statistical measures such as mean and variance. In this article we show that, by treating the fluctuations as a time series, one can apply techniques such as power spectra and rescaled range analysis to gain insight into the mechanisms underlying the remarkable abilities of humans to perform a variety of rhythmic movements, from maintaining memorized temporal patterns to anticipating and timing their movements to predictable sensory stimuli. © 2001 Elsevier Science

INTRODUCTION

The human motor system is often required to act in a specific timing relationship with certain external rhythmic events. Despite a venerable history of investigation, it is not well understood how this feat is achieved. A basic experiment to study this behavior asks the subject to tap in synchrony (on the beat) or in syncopation (off the beat) with a periodic stimulus. Although the subject can perform the task well for a range of frequencies, inevitable timing errors occur on each tap, the analysis of which is an important step toward unraveling the underlying control mechanism.

Dunlap (1910) was the first to carry out a systematic study of the synchronization experiment. One of his main observations was that the mean synchronization error (asynchrony) is negative, indicating that the subject is not simply reacting to the stimulus but tends to tap in advance of the stimulus onset, in contrast to the title of his article, “Reaction to Rhythmic Stimuli, with Attempts to Synchronize.” This result and its subsequent confirmation led to the Paillard–Frasse hypothesis, which established the essential role of sensory feedback in the maintenance and execution of the synchronization task (Aschersleben & Prinz, 1995).

Woodrow (1932) examined the effect of stimulus frequency on the accuracy of performance. He found that the relative performance, measured by the ratio between the standard deviation of the synchronization errors and the stimulus period, is an inverse bell-shaped function of the stimulus frequency, with the minimum at 1.25 Hz. This result quantifies the empirical fact that the task becomes more difficult to perform at both the high and the low frequencies.

Mean and variance (standard deviation) are static measures that do not reflect the temporal structure in the data. Michon and van der Valk (1967) were the first to explicitly point out that “studies of tapping behavior have failed to incorporate the

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sequential aspects of the performance of subjects," and proceeded to introduce linear systems theory to model the synchronization experiment. Hary and Moore (1985, 1987) gave sequential correlation a more meaningful role by relating it to the subject's "synchronization strategy." Their model hypothesizes that the time-keeping mechanism switches randomly between the metronome reference frame and the subject's own tap reference frame. Recent work by Vorberg and Wing (1996) and by Pressing and Jolley-Rogers (1997) postulates that the subject makes short-term self-correcting adjustments in response to perceived timing errors. By adapting the Wing-Kristoffer-son (1973a, 1973b) two-component model originally developed for unpaced rhythmic tapping, these authors proposed a first-order autoregressive type of model to describe the temporal dynamics of the timing errors, which gives an exponentially decaying autocorrelation function.

We carried out finger-tapping experiments that produced long stationary fluctuating time series while the subjects performed the tasks of synchronization and syncopation to a regular sequence of sounds. Our examination of the overall correlation structure of the timing errors uncovered a pattern that is very different from that predicted by the self-correcting model. In particular, using an array of diagnostic tools including spectral maximum likelihood estimator and rescaled range analysis, we showed that the timing errors exhibit $1/f^\alpha$ -type power spectra. By virtue of the Fourier transform, the autocorrelation function $C(k)$ scales with the time lag k as a power law $C(k) \sim k^{-\beta}$, where $\beta = 1 - \alpha$. This slow decrease of the correlation between errors separated by the interval k is in contrast with the rapid exponential decay predicted by the self-correction model. In fact, for α between 0 and 1, as is the case in the present experiment, the $C(k)$ function decays so slowly with k that the underlying statistical process is said to have long memory (Beran, 1994). (It is worth noting that the term memory here is used in a statistical sense and does not have any bearing on human memory.) Remarkably, the value of the scaling exponent α allowed us to distinguish clearly the two basic modes of coordination in that different values of α are obtained for synchronization and for syncopation. We further present some evidence that α may be influenced by cognitive strategies employed by the subject in producing syncopation, indicating that the long-term statistics has a higher cortical origin. These results, in conjunction with our understanding of the generating mechanism for $1/f^\alpha$ -type long memory processes, suggest the hypothesis that the brain has distributed timing control structures acting on multiple time scales.

METHODS

Task. Subjects were seated in front of a computer inside a sound-attenuated recording chamber and instructed to rhythmically press their right index finger on a computer key in a specific phase relationship with periodic auditory beeps (duration = 50 ms). A computer program was used to register the time of the key depression with 1-ms resolution. The timing errors, defined as the time between the computer recorded response time and the metronome onset time, are recorded as a time series indexed by the number of taps. All data are based on subjects' successful completion of the requirement without interruption, without missing stimuli or making extra responses.

Five right-handed subjects took part in the first experiment in which each performed two trials of 1200 continuous synchronization taps against the metronome set at 2 Hz. A total of 10 time series were collected.

Eight subjects performed the two conditions in the second experiment where the metronome was set at 1 Hz. In the synchronization condition the subject was required to tap in synchrony with the metronome. The syncopation condition required the subject to press the computer key in between adjacent stimuli. Each subject performed two trials for each condition, and a total of 32 time series, each containing 550 points, were collected from eight male subjects.

The third experiment was designed to produce syncopation by using different coordination strategies. For the "2:1" strategy, subjects ($N = 4$) made two flexion movements during every 1-s stimulus interval,

one flexion movement synchronized with the stimuli (no key depression), and a second key depression movement occurred in the middle of the cycle, thus syncopating with the 1-Hz stimuli. In a way this is like playing a drum twice as fast as the basic musical rhythm. The second strategy required subjects ($N = 4$) to intentionally control an extension movement (key-release) with the stimulus instead of a downward flexion key-press, so that when the finger moves back to the lowest position, the (computer-registered) key-press was in the middle of the cycle (syncopation). At 1 Hz, all these modes of coordination can be maintained, thus giving us a unique opportunity to investigate changes in the coordination dynamics even though the environmental conditions remained the same. A total of 16 time series, each again containing 550 points, were collected in the third experiment.

Data analysis. For all time series, the first 10 data points were eliminated to remove initial transients. The next 1024 points from the first experiment and the next 512 points from the second and third experiments were considered as realizations of stationary stochastic processes and used for further analysis. The remaining data points in each time series are discarded.

A stationary stochastic process is characterized by its mean, variance, and autocorrelation function $C(k)$. The Fourier transform of $C(k)$ gives the power spectrum. In this work the power spectrum is estimated using the periodogram method after removing the mean and normalizing by the standard deviation.

The autocorrelation function generally decays to zero as k increases. If $C(k)$ decays sufficiently slowly such that

$$\sum_{k=0}^{k=\infty} C(k) = \infty, \quad (1)$$

we say that the process has long memory. Specifically, this occurs when

$$C(k) \sim k^{-\beta}, \quad (2)$$

with $0 < \beta < 1$. The power spectrum of a long memory process is singular at $f = 0$ and scales with f as $S(f) \sim f^{-\alpha}$, where $\alpha = 1 - \beta$ (Beran, 1994).

To calculate the spectral exponent of each individual time series, the frequency domain maximal likelihood estimation (MLE) was used, which estimates the long memory in a time series from its power spectrum based on a fractional Gaussian noise model. The details of the method and the mathematical background can be found in Beran (1994).

Another index for long memory processes is the Hurst exponent H . It relates to α through (Beran, 1994)

$$H = (1 + \alpha)/2 \quad (3)$$

A direct way to estimate the value of H is the trend-corrected rescaled range analysis (Beran, 1994). Let e_i be the i th timing error and $\bar{e}_i = e_i - \bar{e}$, where \bar{e} denotes the sample mean of the error time series. Consider the cumulative sum, $L(n, s) = \sum_{i=1}^s \bar{e}_{n+i}$, where $L(n, s)$ can be regarded as the position of a random walk after s steps. Define the trend-corrected range of the random walk as $R(n, s) = \max\{L(n, p) - pL(n, s)/s, 1 \leq p \leq s\} - \min\{L(n, p) - pL(n, s)/s, 1 \leq p \leq s\}$. Let $S^2(n, s)$ denote the sample variance of the data set $\{\bar{e}_{n+i}\}_{i=1}^s$. If the average rescaled statistic $Q(s) = \langle R(n, s)/S(n, s) \rangle_n$ scales with s as a power law for large s , $Q(s) \sim s^H$, then H is the Hurst exponent. One can show that, if the autocorrelation function $C(k)$ sums to a finite number, then generally $H = 1/2$ (Rangarajan & Ding, 2000), corresponding to the case of short term memory. If Eq. (1) holds, then $1/2 < H < 1$, and the time series is said to have long persistent memory. It is worth pointing out that the spectral method or the rescaled range method applied alone can be susceptible to producing false results. Here we employed both methods in an integrated approach (Rangarajan & Ding, 2000), which requires the outcomes from the two methods to be consistent through Eq. (3).

RESULTS

Experiment 1. Figure 1 shows a typical example of an error time series. Highly irregular fluctuations indicate the presence of a underlying random process. We computed its power spectrum, which is shown on a log-log scale in Fig. 2a. Excellent straight line fit means that the power $S(f)$ scales with the frequency f as a power law $S(f) \sim f^{-\alpha}$, where $\alpha \approx 0.54$. The rescaled range plot for the same data is shown in Fig. 2b where a straight fit gives $H \approx 0.79$, which is consistent with the spectral α value through Eq. (3). From all 10 time series we obtain $\alpha = 0.49 \pm 0.02$.

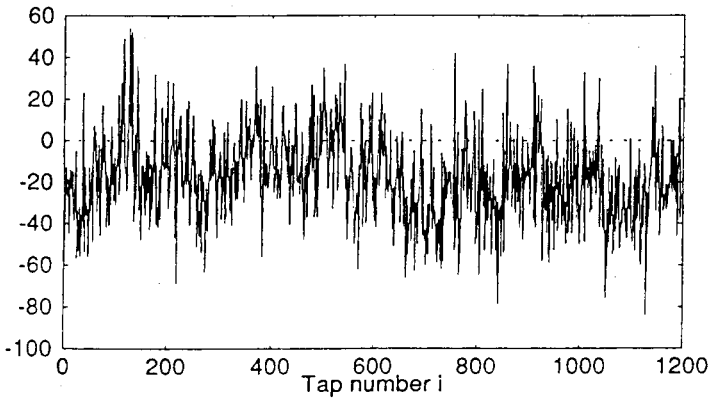


FIG. 1. Example synchronization error time series from the first experiment. The error is defined as the timing difference between a key-press and the associated metronome onset measured in ms resolution.

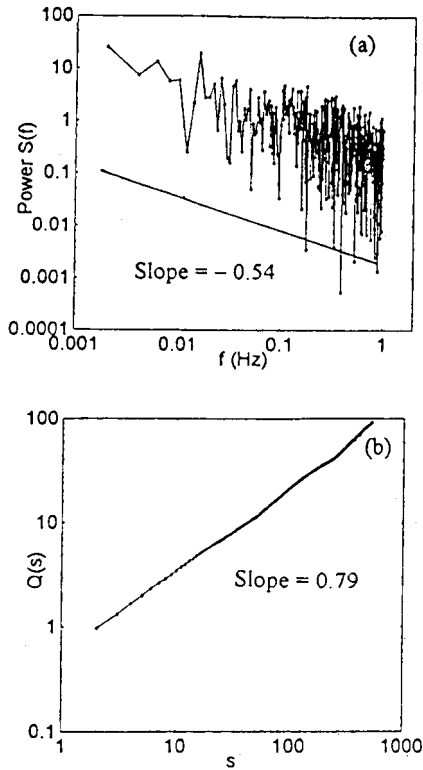


FIG. 2. (a) Spectral density of the error time series in Fig. 1. We have converted the unit of frequency from 1/beat to Hz in the following way. Because the interval between two data points in error time series is the stimulus period T (0.5 s in this case), the frequency f is define as $f_i = (k - 1)/(mT)$, for $k = 1, 2, \dots, m/2 - 1$, where $m(1024)$ is the total length of the time series. (b) Log-log plot of averaged R/S value $Q(s)$ against window size s for the time series in Fig. 1. Adapted from Chen, Ding, & Kelso (1997).

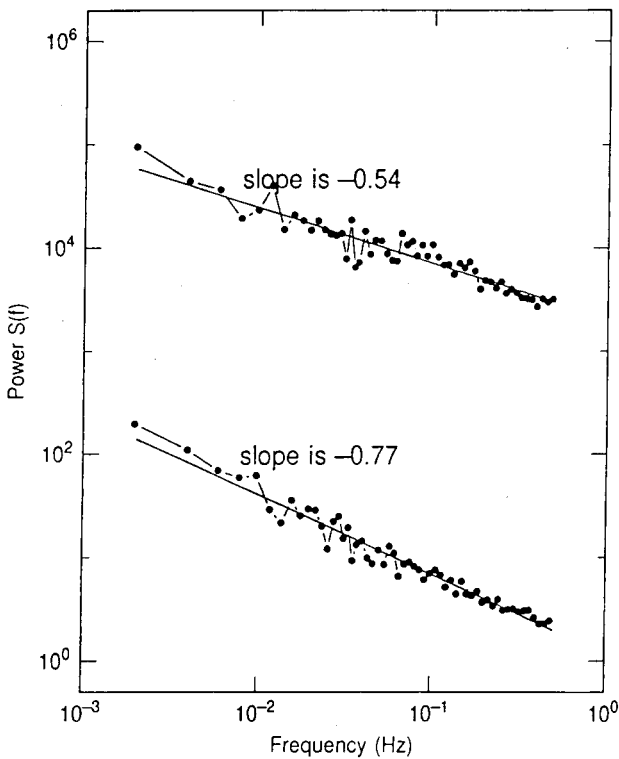


FIG. 3. Averaged spectral density of the error time series for synchronization (top) and syncopation (bottom) from the second experiment.

Experiment 2. Figure 3 shows the averaged power spectra for the two basic conditions in a log–log plot. The scaling exponents are estimated to be $\alpha = 0.54 \pm 0.03$ for synchronization (top) and $\alpha = 0.77 \pm 0.02$ for syncopation (bottom). Although the averaged spectra appear to give different slopes for the two movement conditions, to evaluate the statistical significance of the difference, we applied the frequency domain MLE to each of the 32 time series (16 from synchronization and 16 from syncopation) to calculate the individual exponent α . The overall α value for synchronization is 0.45 ± 0.16 , while for syncopation, $\alpha = 0.73 \pm 0.17$. The two exponents are significantly different by a two-tailed t test ($p < .025$).

Experiment 3. Synchronization is a relatively simple task for humans to perform. Syncopation, however, is more demanding, requiring longer training and a higher level of concentration to ensure quality performance. Moreover, it is known that syncopation is a less stable form of coordination, and that spontaneous switching to synchronization can occur at both behavioral (Kelso, DelColle, & Schner, 1990) and neural levels (Kelso et al., 1992). The difference between the α values for the two conditions suggests that the scaling exponent is a correlate of task difficulty. It is known that syncopation (e.g., flexion movements off the beat) can be performed at high movement rates if the subject consciously decides to place extension movements on the beat (Kelso et al., 1990). The question is whether it is possible to shift the values of the scaling exponents by altering cognitive strategy. Were this so, long-range correlation in timing behavior may be said to originate in the higher level functioning of the human brain. The third experiment was designed to probe this idea, and required subjects to use different sensorimotor coordination strategies to produce movement responses between stimuli, thus giving rise to variable timing

errors mimicking the syncopation task. For the 2:1 strategy, using MLE, the scaling exponent from the eight time series was found to be $\alpha = 0.48 \pm 0.19$, which is nearly the same as that of the original synchronization. For the subjects who took part in the “extension on the beep” strategy, the scaling exponents were found to be $\alpha = 0.87 \pm 0.07$, which is very close to the syncopation value. However, closer inspection of each subject’s response indicates that, for one subject who performed the task particularly well, the exponent is about 0.45, close to that of the synchronization task. These results offer some evidence that the cognitive strategy may influence the value of the scaling exponent α , lending support to the notion that the source of long-range correlations in timing lies in the central nervous system.

DISCUSSION

Recently proposed ideas (Vorberg & Wing, 1996; Pressing & Jolley-Rogers, 1997) assert that the timing error e_n obeys the first-order autoregressive [AR(1)] model

$$e_n = \lambda e_{n-1} + \varepsilon_n, \tag{4}$$

where ε_n is a uncorrelated random process. Equation (4) is a result of modifying the central clock component of the Wing–Kristofferson model to include the influence of sensory feedback. According to this equation, the autocorrelation function of the errors decays exponentially in the form of $C(k) \sim \lambda^k$, leading to a Lorentzian type of power spectrum shown in Fig. 4 (top curve). The inconsistency between this prediction and the power law spectrum (bottom curve) is apparent.

The exponentially decaying autocorrelation function defines a characteristic time scale in terms of the correlation length, which is a function of λ . A long memory process is a process that does not possess a single time scale. In recent years, a number of mechanisms, ranging from self-organized criticality (Bak, Tang, & Wiesenfeld, 1987) to multiscaled randomness (Hausdorff & Peng, 1996), have been proposed to explain the occurrence of such processes. A central theme of these models is the presence of many interacting components acting on different time scales. Here

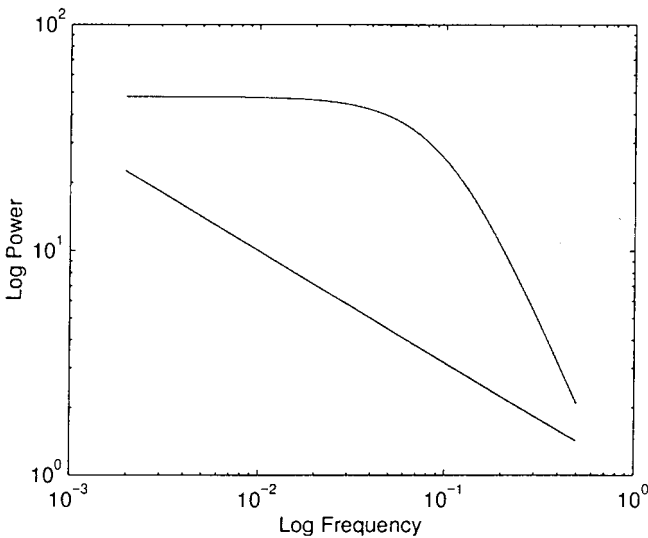


FIG. 4. The power spectrum of a first-order autoregressive process (top) is plotted against a $1/f^\alpha$ -type spectrum (bottom). Notice that in the low frequency portion the autoregressive process shows a flattened spectrum.

we mention an explicit mathematical model of long memory processes proposed by Granger (1980). The discussion of this model is instructive since the basic building block in the model is an AR(1) process like Eq. (4) and it may lead to fruitful speculations on the origin of the observed $1/f^\alpha$ spectra.

Let $x_n(j)$, $j = 1, 2, \dots$, be independent AR(1) processes defined by

$$x_n(j) = \lambda_j x_{n-1}(j) + \varepsilon_n(j), \quad (5)$$

where $-1 < \lambda_j < 1$, and $\{\varepsilon_n(j)\}$ is a zero mean random process with variance σ_j^2 . Consider the sum of N such processes

$$x_n^{(N)} = x_n(1) + x_n(2) + \dots + x_n(N). \quad (6)$$

Granger (1980) showed that, if λ_j and σ_j^2 are randomly and independently drawn from a beta distribution with suitable characteristics, then for large N , $x_n^{(N)}$ is a process of long memory and exhibits $1/f^\alpha$ spectra. Numerical simulations (Rangarajan & Ding, 2000) show that, for a finite data set of several hundreds points, the sum of 4 or 5 AR(1) processes can behave in a way similar to the experimental observation reported here.

Many lesion and pharmacological studies have revealed diverse brain structures having time-keeping functions. For example, the basal ganglia is shown to be important in rhythmic movement (Freeman, Cody, & Schady, 1993), because Parkinsonian patients have a great deal of difficulty in performing various timing tasks. Ivry and colleagues (Ivry & Keele, 1989) studied motor timing in patients with cerebellar atrophy and showed elevated variability in the clock component. Recent brain imaging work has begun to reveal cortical structures that are important in the control of timing behavior (Harrington & Haaland, 1999; Chen, Ding, & Kelso, 1999; Muller et al., 2000). We suggest that the $1/f^\alpha$ long memory characteristic of the error time series is consistent with the view that distributed neural systems participate in the maintenance and execution of the synchronization and syncopation tasks. Furthermore, we hypothesize that each participating system may have its own time-keeping mechanism, which undergoes Eq. (4) type of self-correction at different time scales.

Dunlap (1910) was the first to observe cycling behavior in the error time series. He noted that the errors tend to grow larger and larger in a given direction (either early or late relative to the stimulus) until a correction causes a change in direction where the same pattern repeats. Dunlap called this the drifting phenomenon and attributed it to a frequency mismatch between stimulus and response. Spectral analysis reported here reveals a highly organized power law relationship between the amplitude of the cycle and its frequency.

A very similar revelation has taken place in the past few years in a related movement experiment called the continuation task, in which subjects continue to reproduce a given time interval from memory after the metronome has been turned off. Early work on this paradigm concentrated on the correlation between two adjacent intertapping intervals (Wing & Kristofferson, 1973a, 1973b). Later experiments found that intervals not directly abutting each other also exhibited significant correlation (Wing, 1977). More recent work using spectral techniques shows that in fact the interval can be described by a $1/f^\alpha$ process with $\alpha = 1$ and such a process may take on important cognitive functions (Gilden, Thornton, & Mallon, 1995).

Finally, we note that, despite the fact that $1/f^\alpha$ processes have been observed in many physical and biological systems (Bak et al., 1987; Bassingthwaite, Liebovitch, & West, 1994; Chen, Ding, & Kelso, 1997; Collins & De Luca, 1995; Peng, Havlin, Stanley, & Goldberger, 1995), the functions of such processes have remained obscure. Our experiments establish a functional role for these types of processes by

showing that the scaling exponent differentiates basic modes of sensorimotor coordination and may be altered by cognitive manipulations.

CONCLUSION

We established that the timing fluctuations in human sensorimotor coordination tasks are characterized by a $1/f^\alpha$ type of long memory process. The specific value of the exponent α was determined by task difficulty (synchronization or syncopation) and may be altered by coordination strategy. Taken together, our experiments suggest that the source of $1/f^\alpha$ fluctuations in human sensorimotor coordination lies in the multiple time scale activities of distributed neural areas in the human brain.

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