FREIGHT MOBILITY RESEARCH INSTITUTE
College of Engineering & Computer Science
Florida Atlantic University

Project ID: Y2R8-18

DYNAMIC TRAJECTORY CONTROL AND SIGNAL COORDINATION FOR A SIGNALIZED ARTERIAL WITH SIGNIFICANT FREIGHT TRAFFIC

Final Report

by

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May, 2021

ACKNOWLEDGEMENTS

This project was funded by the Freight Mobility Research Institute (FMRI), one of the twenty TIER University Transportation Centers that were selected in this nationwide competition, by the Office of the Assistant Secretary for Research and Technology (OST-R), U.S. Department of Transportation (US DOT).

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EXCUTIVE SUMMARY

This project considers the problem of improving mobility by controlling multiple truck trajectories along multiple intersections and improves the mobility of traffic flow by optimizing local travel time and smoothing trajectory. The proposed method is built in a connected and automated vehicle environment.

The problem is addressed at two levels, including individual vehicle level(lower) and strategical (upper) level. At the individual vehicle level, we first consider the problem that an individual truck traveling within one block between two intersections, and then we scale the problem towards multiple vehicles along consecutive intersections. For the simplification purpose, this problem was efficiently decomposed into four scenarios according to the physical constraints of vehicles and the relation between arrival time and traffic signal information. In each scenario, to smooth trucks' trajectories, practical optimal travel time was locally obtained, and a constrained optimal control was introduced to plan individual trucks. At the strategical level, to scale the problem from one intersection to multiple consecutive intersections, instead of applying a centralized approach, a decentralized control was applied. The method calculates local practical travel time and solves constrained optimal control problems sequentially.

Numerical experiments with two baselines were developed to compare with the proposed method. The experiments suggested that the method significantly reduces travel time by reducing stops and smoothing trajectories. Baseline 1 is with a non-CAV setting when all trucks were human-driven vehicles. Compared with baseline 1, the proposed method could save travel time up to 26.85 % while traffic demand is high and up to 29.33 % while demand is moderate. Baseline 2 is a setting when only a trajectory smooth method is applied at an isolated intersection without consideration of optimal local travel time. Compared with baseline 2, the proposed method can save travel time up to 60.36% when the demand is low and up to 56.26 % when demand is high.

1. INTRODUCTION

Freight mobility in an urban area has become a challenging topic that attracting researcher's eyes. Improving mobility where freight traffic is significant is a major concern recently. However, truck drivers usually experience stop-and-go waves traveling through signalized intersections when most of the surrounding vehicles are driven by humans. Traffic oscillation and queue backpropagate may result in a capacity drop, leading to an increase in travel time and a decrease in mobility (Treiber et al., 2006). On an urban street, even when the signals are well-coordinated, the travel time increase for drivers traveling through consecutive signalized intersections, since the dispersions of vehicle platoons may lower the performance of the coordination of signals (Denney Jr, 1989). Systematic methods for controlling trucks in an urban area are essential.

For a single CAV, the car-following in a platoon or lane choices were not considered, which should be considered when a platoon of CAV was involved in the problem. Although the problem about longitudinal control of connected automated vehicles has been widely studied, the control for CAVs is hard when considering consecutive signalized arterials, which can lead to a problem of variable control horizon. Besides, the synchronization of calculation of CAV travel time and trajectory is a difficulty in the proposed problem. To fill in the gap, this project provides an approach of hierarchical longitudinal control, tackle the variable horizon of CAVs, and give insight into the scenarios of CAV control on a signalized corridor. A centralized method is unable to model HVs, which are uncontrollable.

Concerning the problem of longitudinal control strategies of vehicles in urban areas, at least one signalized intersection is modeled. Many previous studies concerned the strategies for vehicles approaching an isolated intersection. For instance, Rakha and Kamalanathsharma (2011) developed eco-driving strategies for vehicles at an isolated intersection by integrating microscopic fuel consumption models in objective functions to minimize environmental adverse effects(Rakha and Kamalanathsharma, 2011). They also proposed a dynamic programming-based method to control the speed of a vehicle by splitting the process of approaching a signalized intersection into three states, showing that the method can save fuel and travel time significantly for an individual vehicle(Kamalanathsharma and Rakha, 2013). Yang et al. (2016) developed an Eco-CACC system to improve the fuel efficiency of CAVs at an isolated intersection considering existing queues. Optimal control is used to design trajectories for leading CAVs of platoons, to lead vehicles smoothly approaching an isolated intersection. The performances under different market penetration rates are demonstrated, showing a throughput benefit ranging from 0.88% to 10.80% (Jiang et al., 2017). A shooting heuristic (SH) is proposed for optimal control solutions for vehicle trajectories at intersections (Ma et al., 2017; Zhou et al., 2017). Individual Variable Speed Limits with location optimization are designed to smooth trajectories of CAVs to improve mobility at an intersection (Yao et al., 2018).

Except for studies only considering one intersection model, more pieces of literature study control strategies for consecutive traffic signals since the traffic signals are usually configured consecutively along the roadway in urban areas. Mandava et al. (2009) applied a dynamic speed-advise method to drive a CAV smoothly along consecutive intersections when no surrounding vehicles are concerned (Mandava et al., 2009). The method reduced fuel consumption, CO2 emissions significantly, and travel time slightly (1.06%) for a single vehicle. Barth et al. (2011) developed an optimal control for a single vehicle to drive along consecutive signalized intersections, with a reduction in fuel consumption and CO2 emissions. Other than the reduction in environmental adverse effects, queue minimization is considered in the development of the optimal trajectory of one single vehicle along consecutive intersections , which leads to an additional delay for the following vehicles (He et al., 2015). A mix-integer programming sequential convex optimization is used to design an optimized speed plan of a vehicle when traveling along signalized intersections, saving travel time up to 6.00% (Huang and Peng, 2017). Tang et al. 2018 incorporated a speed strategy into a car-following model for multiple vehicles to pass multiple intersections (Tang et al., 2018).

It was important to improve the mobility and traffic performances of CAVs in an urban environment with signals. Although proposing methods to control CAVs at intersections was popular, previous studies focused on the problem, either at one single intersection or for one single vehicle. Others considered CAVs when no HVs were assumed. Even when HVs were considered, the combination structure of mixed traffic needed to be assumed; for example, (Zhao et al., 2018) used scenarios in the experiment to show the possible combination of HVs and CAVs. To our best knowledge, few have studied a general issue in which multiple vehicles passing consecutive signals, considering a mix of CAVs and HVs with an arbitrary combination. Considering the general problem is crucial and necessary. Since although smooth trajectories could reduce travel time by reducing time-consuming stop and startup driving behaviors at an intersection, it may also lead to an increase in travel time if it was not considered systematically (He et al., 2015). What is more, some underly relations in the problem will be revealed only when all the factors were considered.

Therefore, our research proposed a method to investigate a more general problem with fewer assumptions than previous work by modeling an individual vehicle traveling within one block and scaling towards multiple vehicles along consecutive intersections. The research has lead to a publication in (Xiao et al., 2021). The relations among vehicles and the relations between vehicles and the signals were concerned simultaneously. The method is implemented in a decentralized fashion. Like most similar studies, to focus on longitudinal behaviors the problem is considered on a single lane. The contributions are:

- To propose a systematic method to analyze CAVs at signals based on split scenarios according to preceding vehicle and signal conditions.
- Develop a hierarchical longitudinal control for CAVs considering variable horizon optimal control in urban streets.

2. METHODOLOGY

The problem that an individual vehicle traveling within one block between two intersections is introduced in this chapter and then the problem is scaled towards multiple vehicles along consecutive intersections in the next chapter. To analyze the problem, firstly, the constraints of the longitudinal position and travel time of a vehicle are mathematically described. Then the constraints are decomposed into different sets. Each set corresponds to a scenario. Finally, each scenario is explained with their transportation meaning and provided a solution to find minimum practical travel time. As shown in Figure 1, the mixed traffic travels through consecutive intersections on the urban street from intersection 1 at upstream 1 to intersection i at downstream. The traffic is a mixture of HVs and CAVs. Communication devices are installed on CAVs to ensure real-time information exchange via V2V and V2I.

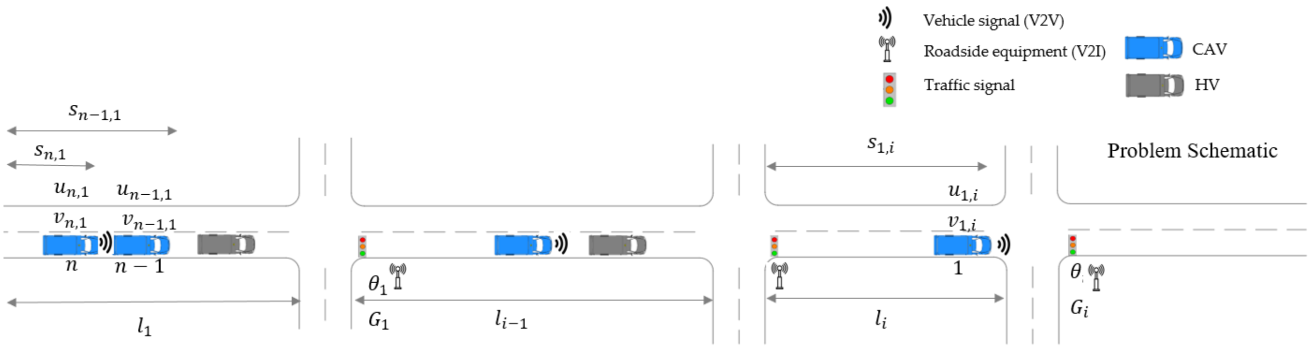

Figure 1 The schematic of the problem

The assumptions are listed here: The V2V communication is assumed to be active once a vehicle entering the block. Information related to the timing plan such as offset θ_i, the duration of green G_i, green elapse time $G_{n,i}$, and geometrical variable block length l_i can be received by CAVs with no delay. Like previous studies, a single lane setting is assumed so the overtaking behavior of a vehicle is not in the scope of concerns. The car following behaviors of HVs are assumed as known and HVs slow down and stop in front of a signal when they cannot pass within the current green.

In Figure 1, the vehicles move forward in their longitudinal direction and time coordination. The travel time of a vehicle within a block is defined as the duration between the time instant when it passes the intersection $i-1$ and the time instant it passes intersection i.

For individual vehicles, the trajectory generates in an optimal control fashion. The state $x_{n,i}$ of a vehicle n in intersection i is defined as a combination of its longitudinal position $s_{n,i}$ within this block i and longitude speed $v_{n,i}$:

$$x_{n,i} = \left(s_{n,i}, v_{n,i}\right)^T \tag{1}$$

The vehicle dynamics within a block for a CAV are expressed by a state-space representation, indexed by the number of vehicles and the intersections. On an urban street, the vehicles are not allowed to move backward. A CAV can obtain information of vehicle status such as position, acceleration, and speed from the preceding vehicle, no matter whether the preceding vehicle is a CAV or an HV.

Table 1 Major notations in this report

Notation	Description
$t(sec)$	Time instant
n, i	The subscript of vehicle n and intersection i
x	State of a vehicle
$s(m), v(m/s), u(m/^2)$	The longitudinal position, speed and acceleration of a CAV
$p(m)$	The longitudinal position
$v^*(m/s)$	The targeted speed at the end of this control horizon
$l^*(m)$	The targeted distance traveled at the end of this control horizon
$v_0(m/s)$, $u_0(m/s^2)$	Desired speed; desired acceleration
$l_i(m)$	The length of the block between intersection $i-1$ and intersection i
J	Cost function of the optimal control
$t_k(sec)$	Starting time of an optimal control horizon
$T(sec)$	Terminal time of an optimal control horizon
L	The running cost
Φ	The terminal cost
$\theta_i(sec)$	Offset at intersection i with respect to intersection i - 1
$G_i(sec)$	Green time at intersection i
$C(sec)$	The cycle of a signalized intersection
$t_{0,n}(sec)$	The desired headway for vehicle n and vehicle $n-1$
k	The cycle multiplier
k^*	The optimal cycle multiplier
$M_{n,i}(sec)$	The moment vehicle n passes intersection i
$T_{n,i}(sec)$	The travel time spent by vehicle n in the block towards intersection i
$T_{n,i}{}^{(f)}(sec)$	The duration that the subject CAV n following its preceding CAV within this block i
$G_{n,i}(sec)$	The green elapse time when vehicle n enters block i
z_n	All the decision variable of vehicle n
I_0, I_1, I_2, I_3	The number of intersections that a CAV is planned as scenario 0, 1,2,3

The research question is how to reduce the travel time for all vehicles when they are traveling from the first intersection to the final intersection and provide a suitable trajectory for them. The difficulties of this problem are that: Traffic signals exist along consecutive intersections, cutting off the traffic. Multiple statuses exist for a vehicle, in which varying control horizons can appear.

2.1 LOWER-LEVEL CONTROL: MATHEMATICAL FORMULATION OF OPTIMAL CONTROL

The longitudinal control for CAVs follows a hierarchical structure: at the upper level, the travel time is calculated; at the lower level, the optimal control is applied to generate the trajectories.

2.1.1 Optimal control design

When an individual vehicle is traveling within one block between two intersections, its state including position and speed is known. The problem is decomposed into different scenarios and is then scaled towards multiple vehicles along consecutive intersections. The constraints from the longitudinal position and feasible arrival moments of a vehicle with the presence of signals are mathematically described. Each scenario is explained with their transportation meaning and provided a solution of minimum travel time and trajectory.

The system writes with a linear time-invariant system (LTI):

$$x_{n,i}(t) = Ax_{n,i}(t) + Bu_{n,i}(t) \tag{2}$$

$$\mathrm{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \mathrm{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{3}$$

where the control variable $u_{n,i}$ is the acceleration of the vehicle. The cost function to ensure optimal performances is defined as follows considering the comfort and terminal performances:

$$J_{n,i} = \min \int_{t=0}^{T_{n,i}} L\left(x_{n,i}(t), u_{n,i}(t)\right) dt + \Phi\left(T_{n,i}, x_{n,i}(T_{n,i})\right) \tag{4}$$

where the ending time or the control horizon $T_{n,i}$ is a variable which is determined systematically. It is then discussed in 3.2, based on different scenarios. The running cost is set as an instantaneous cost showing the penalties concerning comfort. It is expressed as the quadratic term of acceleration.

$$L = \frac{1}{2} {u_{n,i}}^2 \tag{5}$$

The terminal cost gives penalties so that the final states can approach desired values (terminal speed and terminal distance).

$$\Phi = w_1\left({x_{n,i}}^{(1)}(T_{n,i}) - {l^*}_{n,i}\right)^2 + w_2\left({x_{n,i}}^{(2)}(T_{n,i}) - {v^*}_{n,i}\right)^2 \tag{6}$$

Again, $T_{k,i}$ will be determined systematically. Weighing factors w_1 and w_2 show the penalty for the state deviation from the terminal speed and the terminal distance at the end of the horizon.

2.1.2 Optimal control solution

The desired speed is set the terminal speed at each intersection for each vehicle $v^*_{n,i} = v_0$. The block length between two intersections is used as terminal distance $l^*_{n,i} = l_i$.The problem then writes:

$$J_{n,i} = \sum_{K=1}^{T} \left({u_{n,i}}_{t+k-1}{}^2\right) + w_1 \left({x_{n,i}}^{(2)}{}_T{}^2 - 2{x_{n,i}}^{(2)}{}_T v_{n,i}{}^* + v_{n,i}{}^{*2}\right) + w_2 \left({x_{n,i}}^{(1)}{}_T{}^2 - 2{x_{n,i}}^{(1)}{}_T l^* + l_{n,i}{}^{*2}\right) \quad (7)$$

s.t.

$$\left(x_{n,i}, u_{n,i}\right) \in \Omega \cap U \quad (8)$$

Ω represents the constraints from vehicle dynamics, including the limitation from maximal speed, maximal acceleration, distance, etc. U represents the physical constraints from the preceding vehicle during the period when it follows preceding vehicle $f_{n,i}$.

$$\Omega = \left\{{x_{n,i}}_{t+1} = A_d {x_{n,i}}_t + B_d {u_{n,i}}_t, {u_{n,i}}_t \in \left(u_{n,i,lb}, u_{n,i,ub}\right), {x_{n,i}}^{(1)} \in (0, l_i), {x_{n,i}}^{(2)} \in \left(v_{n,i,lb}, v_{n,i,ub}\right)\right\} \quad (9)$$

$$U = \left\{s_{n,i} \leq s_{n-1,i} + d_s + d_v, t \in (0, f_{n,i})\right\} \quad (10)$$

where d_s is a safe distance that can ensure safety and d_v is the vehicle length; $f_{n,i}$ is the duration of following, determined differently in different scenarios in upper-level control.

The linearly constrained LQ (linear quadratic) optimal control problems are converted to discrete versions and solved by quadratic programming. The objective function becomes:

$$J = \frac{1}{2} \sum_{K=1}^{T} x_{t+k}^T Q_{t+k}\, x_{t+k} + u_{t+k-1}^T R_{t+k} u_{t+k-1} + x_N^T Q_N x_N \quad (11)$$

Matrices Q, Q_N and R are diagonal matrices to ensure the positive-definiteness. They are set as

$$Q = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, R = 1, Q_N = \begin{pmatrix} w_1 & 0 \\ 0 & w_2 \end{pmatrix} \quad (12)$$

2.2 UPPER-LEVEL CONTROL: DETERMINATION OF TRAVEL TIME

Having set the variable horizon optimal control, the horizon $T_{n,i}$ is to be determined systematically. Some prerequisites are provided.

2.2.1 Following behavior along consecutive signalized intersections

With the availability of V2I techniques, CAV receives signal information including current state and future time phases such as G_i and θ_i. The arrival moments should be in a feasible region (the collection of green) and the physical constraints should always hold for safety concerns. To avoid stopping, for CAVs, the set of feasible arrival moments $M_{n,i}$ should be in the collection of green time G:

$$M_{n,i} \in [\theta_i + C * (k-1), \theta_i + G_i + C * (k-1)] \tag{13}$$

where C is the cycle length. If no preceding vehicle exists, k is the counter of the cycles after the current cycle in which the vehicle can pass. k^* is the optimal k that minimizes the travel time. If a vehicle is not able to pass within this cycle, it is natural that it passes at the next cycle, only if the preceding vehicle has passed. Generally, k could be 0 or 1 showing whether a vehicle is able to pass at this cycle or the next.

$$k^* = agrmin\, T_{n,i} \tag{14}$$

Accumulative position $p_{n,i}(t)$ of a vehicle n at time t can be denoted as the addition of two parts: the accumulative position along previous blocks from 1 to $i-1$, and the current position $p_{n,i}(t)$ in this block i for vehicle n is:

$$p_{n,i}(t) = \sum_{1}^{i-1} s_{n,i} + s_{n,i}\left(t - \sum_{1}^{i-1} T_{n,i}\right) \tag{15}$$

At time t, the vehicle has two state conditions which is either passed block i or not. When the subject vehicle has a preceding vehicle in the same block, an inequality describes the situation:

$$\sum_{i-1} l_i < p_{n,i}(t) < p_{n-1,i}(t) < \sum_{i} l_i \tag{16}$$

Similarly, when the preceding vehicle is not in the same block, an inequality writes:

$$\sum_{i-1} l_i < p_{n,i}(t) < \sum_{i} l_i < p_{n-1,i}(t) \tag{17}$$

If the subject vehicle has a preceding vehicle in the same block, its duration is constrained by the preceding vehicle. The moments that enter or leaves a block can be calculated from the values of accumulated travel time:

$$M_{n,i} = \sum_{1}^{i-1} T_{n,i} \tag{18}$$

$$M_{n,i+1} = \sum_{1}^{i} T_{n,i} \tag{19}$$

When a vehicle has a preceding vehicle, $f_{n,i}$ stands for the time duration that the subject CAV following its preceding vehicle within this block. This duration is the subtraction of the moment the preceding vehicle leaves this block and the moment when the subject vehicle enters the block:

$$f_{n,i} = M_{n-1,i+1} - M_{n,i} \tag{20}$$

To scale the problem to consecutive intersections, $G_{n,i}$ shows the duration of green before the vehicle passes the intersection at the moment $M_{n,i}$. This variable links the time of trajectories between two intersections.

2.2.2 Truck trajectory scenario development

The continuation of position and speed are addressed by introducing variables such as the cycle length C, green time G_i, green elapse time $G_{n,i}$ and offset θ_i. Each vehicle is planned only once in a block, the moment a vehicle passes the previous intersection becomes the starting moment the vehicle enters the next intersection; the information is indicated with the help of green elapse time. The final status of a vehicle becomes the initial status in the next.

For CAVs, the arrival moments at the stop line of each intersection are estimated ahead. For HVs, the arrival moments are estimated using travel time estimation methods. According to the categories of the estimated arrival moments and whether there is a preceding vehicle, four scenarios can be defined, and they are noted as scenario 0, scenario 1, scenario 2, and scenario 3, respectively.

$$0 < s_{n,i}(t) < l_i < s_{n-1,i}(t); M_{n,i} \in [\theta_i + C*(k-1), \theta_i + G_i + C*(k-1)], k \leq 1 \tag{21}$$

$$0 \leq s_{n,i}(t) < s_{n-1,i}(t) \leq l_i; M_{n,i} \in [\theta_i + C*(k-1), \theta_i + G_i + C*(k-1)], k > 1 \tag{22}$$

$$0 \leq s_{n,i}(t) < s_{n-1,i}(t) \leq l_i; M_{n,i} \in [\theta_i + C*(k-1), \theta_i + G_i + C*(k-1)], k > 1 \tag{23}$$

$$0 < s_{n,i}(t) < l_i < s_{n-1,i}(t)\ ; M_{n,i} \in [\theta_i + C*(k-1), \theta_i + G_i + C*(k-1)], k \leq 1 \tag{24}$$

In the schematic diagrams Figure 2, the blue line shows the estimated trajectory of an HV, a black line shows the preceding vehicle trajectory. A magenta line represents the trajectory of a CAV. $G_{n,i}$ is the duration between the instant of the nearest green time start and the instant of the vehicle passes the corresponded nearest intersection. Different from G_i which shows the fixed green time, $G_{n,i}$ shows the green elapse time when vehicle n enters block i. θ_i is the offset. The planned trajectory is expressed by its decision variable $z_{n,i}(s_0)$. A minimized control horizon T of the optimal control, which is also the estimated travel time, is expected from the analysis of these scenarios using background knowledge from traffic.

When the subject CAV is the leading vehicle in the same block, the way to minimize travel time is to accelerate and maintain its desired speed to travel through the block to pass the intersection (setting the speed limit as the desired speed v_0). The minimal travel time is obtained when subject CAV accelerates to the desired speed and maintains the speed until it passes the signal ahead.

$$T^*{}_{n,i} = \{T_{n,i} | (u = u_0 | v \le v_0), (u = 0 | v = v_0)\} \tag{25}$$

The value of $G_{n,i+1}$ in the next intersection $i+1$ is calculated using travel time $T_{n,i}$ and the value of $G_{n,i}$, θ_i from the last intersection.

$$G_{n,i+1} = G_{n,i} + T_{n,i} - \theta_i \tag{26}$$

For the subject CAV with no preceding vehicle in the same block, when it is not expected to pass the intersection within this cycle, it is planned to pass during the green in the next cycle ($M_{n,i} \in [\theta_i + C * (k-1), \theta_i + G_i + C * (k-1)], k > 1$), via a smooth path without stopping. The corresponding $T^*{}_{n,i}$ for both scenario 1 is calculated by:

$$T^*{}_{n,i} = \theta_i + C * k^* - G_{n,i} + G_{n,i+1} \tag{27}$$

$G_{n,i+1}$ varies the arrival moments, which is set as small as possible so that the startup time can be saved compared to human driving behavior.

For scenario 2, the calculation of $T^*{}_{n,i}$ and $G_{n,i+1}$ is the same as that of scenario 1. The difference is the subject vehicle has constraints from its preceding vehicle for the preceding vehicle is in the same block. U is active as the physical constraints of the optimal control.

Scenario 3 shows when the subject CAV follows a preceding vehicle in this intersection, and it passes within the same green window as the preceding vehicle: $M_{n,i} \in [\theta_i + C * (k-1), \theta_i + G_i + C * (k-1)], k \le 1$.The corresponding $T^*{}_{n,i}$ is then calculated from:

$$T^*{}_{n,i} = max(T_{n-1,i} - f_{n,i} + t_{0,i}, \frac{l_i}{v_0}) \tag{28}$$

$$G_{n,i+1} = G_{n,i} + T_{n,i} - \theta_i - C * k^* \tag{29}$$

Note that the minimal travel time cannot be smaller than the value when the vehicle is traveling with the desired speed (in that case, the travel time from scenario 3 is no smaller than that from scenario 0). U is active as the physical constraints from the preceding vehicle.

Although an HV cannot respond to a CAV, a CAV can detect the position of its preceding HV. An estimation of the HV's travel time is conducted. The desired headway $t_{0,i}$ when a CAV following an HV is set to be larger than that an HV follows an HV to ensure safety. The travel time when a CAV follows an HV is calculated as:

$$T^*{}_{n,i} = max(T_{n-1,i} - f_{n,i} + t_{0,i(HV)}, \frac{l_i}{v_0}) \tag{30}$$

An HV is expected to slow down and stop if it cannot pass an intersection within the green duration. They will be modeled remaining standstill at the stop bars during the red phases. The subject CAV does not need to follow closely to an HV. Instead, it passes with a smooth trajectory without stopping. The calculations of $T^*{}_{n,i}$ and $G_{n,i+1}$ are the same as the case when it follows a CAV.

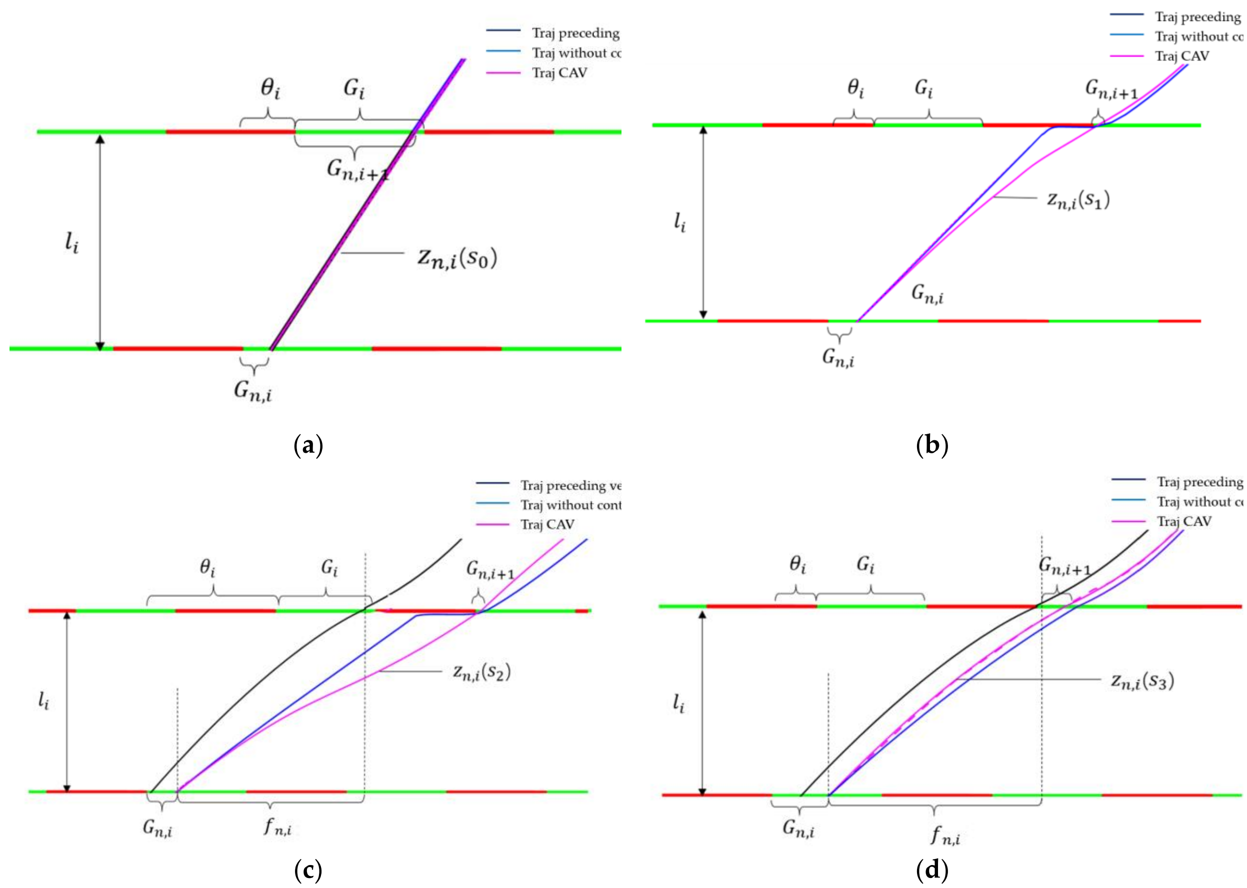

Figure 2. Schematic diagram of (a) scenario 0; (b) scenario 1; (c) scenario 2; (d) scenario 3.

2.2.3 Decentralized control heuristics

Suppose we have N vehicles and I intersections in our evaluation period. A centralized formulation to minimize total travel time for all vehicles can be written as:

$$\min \sum_{n=1}^{N} \sum_{i=1}^{I} \boldsymbol{\Lambda} \times \ \boldsymbol{T}^{*}{}_{n,i} \tag{31}$$

$\boldsymbol{\Lambda}$ is a $N \times I$ matrix with elements 0 or 1, indicating which scenario a vehicle is in.
$\boldsymbol{T}^{*}{}_{n,i}$ is a matrix indicating practical travel time.

Both $\boldsymbol{\Lambda}$ and $\boldsymbol{T}^{*}{}_{n,i}$ are unknown in advance, which makes the problem difficult to solve directly. However, we can obtain the elements of $\boldsymbol{\Lambda}$ and the value of $\boldsymbol{T}^{*}{}_{n,i}$ by exploring. We assume the structure of $\boldsymbol{\Lambda}$ will not change significantly with $\boldsymbol{T}^{*}{}_{n,i}$. So, we can solve this problem by finding minimal practical travel time from vehicle 1 to vehicle N, each vehicle from intersection 1 to intersection I sequentially.

We have information that: The practical travel time calculated from scenario 0 is always the minimum one among all four scenarios. According to Proposition 1, $\sum_{i=1}^{I} T^*{}_{n,i}$ was determined for scenario 1 and scenario 2 if k is determined. According to Proposition 2, the minimization of k^* and $T^*{}_{n-1,i}$ will automatically lead to the minimization of $T^*{}_{n,i}$ for scenario 3. Therefore, we use the propositions as a heuristic to determine $\mathbf{\Lambda}$: At each intersection i, we will assign the problem as scenario 0 or scenario 3 as possible, otherwise with scenario 1 or 2 with minimized k^*.

We define $z_{n,i} \triangleq [u_{n,i}(0)^T, \dots, u_{n,i}(t-1)^T]^T$ as the decision variable of vehicle n from time instant 0 to t in each intersection i. Practical travel time $T_{n,i}{}^*$ is used as an input for trajectory smooth problem. A centralized formulation to smooth the trajectory to optimize their performances (comforts /safety/emissions etc.) can be written as:

$$\min_{z_{n,i}} \sum_{n=1}^{N} \sum_{i=1}^{I} \int_{t=0}^{T_{n,i}{}^*} J(z_{n,i})dt \,, n \in N, i \in I \tag{32}$$

Although we can obtain a globally optimal solution by solving this, there were some difficulties to solve it directly. Firstly, the travel time is unknown and difficult to obtain ahead. Secondly, the constraints are complex and large. Thirdly, the constraints are unknown and dependent at the beginning. Therefore, to synchronize with the calculation of travel time, we decompose the centralized formulation into decentralized optimal controls, and we solve the problem sequentially as well:

$$\min_{z_{n,i}} \int_{t=0}^{T_{n,i}{}^*} J(z_{n,i})dt \tag{33}$$

Each optimal control problem is exactly the problem considered in one of the scenarios. Compared to the centralized problem, the subproblem is easy to obtain optimal control horizon for they can be obtained by exploring: Once the selection of scenarios is made, the practical travel time $T_{n,i}{}^*$ can be calculated. The decision variables of the preceding vehicle are assumed to be known, which is denoted as $\bar{z}_{n-1,i}$.The constraints can be obtained from the results of the last calculation. The decentralized formulation has fewer constraints and decision spaces; therefore it leads to low computation costs. What is more, the structure also makes the method extendable to involve an arbitrary mix of CAVs and HVs.

Suppose we have N vehicles and I intersections in our evaluation period. A centralized formulation to minimize total travel time for all vehicles can be written. To minimize the total travel time is to minimize the summation of travel time of each vehicle in the system:

$$\min_{z_{n,i}} \sum_{n=1}^{N} \sum_{i=1}^{I} T_{n,i} \,, n \in N, i \in I \tag{34}$$

HVs are uncontrollable, their travel time cannot be minimized through the control of trajectory. Suppose m out of n vehicles are CAVs and the rest are HVs, the travel time is separated into two parts denoting the travel time of CAVs and HVs:

$$\min_{z_{n,i}} \sum_{n=1}^{N} \sum_{i=1}^{I} T_{n,i} = \min \underbrace{\left(\sum_{i=1}^{I} T_{1,i} + \sum_{i=1}^{I} T_{2,i} + \cdots + \sum_{i=1}^{I} T_{m,i} \right)}_{CAVs} + \underbrace{\left(\sum_{i=1}^{I} T_{m+1,i} + \cdots \sum_{i=1}^{I} T_{m+2,i} + \sum_{i=1}^{I} T_{n,i} \right)}_{HVs} \quad (35)$$

If an HV is following a CAV, its travel time can be compromised as a function of the CAV selfishly slowing down. However, the travel time of HV is impacted by travel time for CAVs, since HVs will be shaped by CAVs to better trajectories, which has been shown in some previous studies such as (Jiang et al., 2017). Therefore, an assumption is that to minimize the CAVs term will lead to a reduction in the total travel time, which will be validated in the experiments.

The next step is to make sure travel time is minimized each single CAV. For vehicle n that is a CAV, the travel time writes:

$$\sum_{i=1}^{I} T_{n,i} = \sum_{1}^{I_0} {T^*}_{n,i_0} + \sum_{1}^{I_1} {T^*}_{n,i_1} + \sum_{1}^{I_2} {T^*}_{n,i_2} + \sum_{1}^{I_3} {T^*}_{n,i_3} \quad (36)$$

Assuming each vehicle must pass through all the intersections, the number of intersections each scenario (the number of intersections assigned as scenario 0 is marked I_0 , similar for I_1, I_2 and I_3) assigned add up to the number of total intersections:

$$I_0 + I_1 + I_2 + I_3 = I \quad (37)$$

The process of one single CAV traveling through consecutive intersections can be seen as finding a path in a directed graph $G = (V, E)$. In the directed graph, V is a set of nodes showing the scenario options. E is a set of directed edges, which is determined by the feasibility whether a vehicle can drive with a certain scenario. Each edge is with the cost $c(e)$, showing the minimal travel time cost with each scenario at a specific intersection for this vehicle.

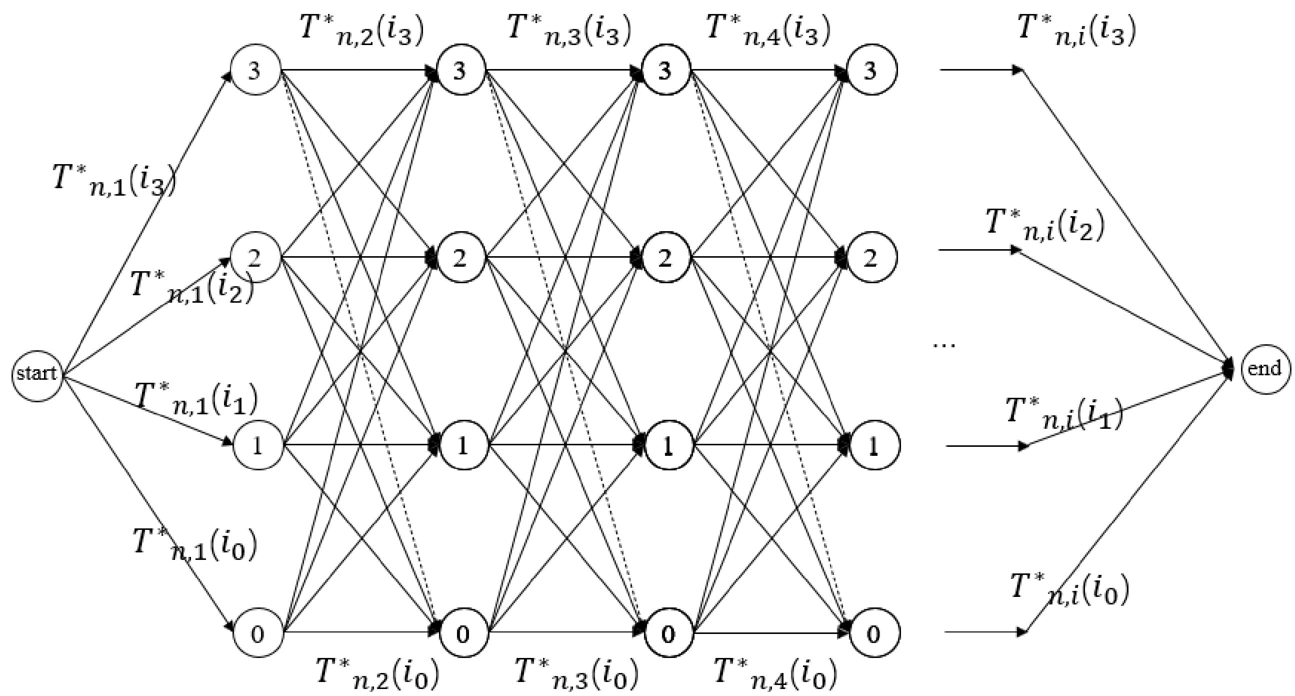

Figure 3 Schematic diagram of scenario 0-3

According to the analysis of scenarios, scenario 0 is that vehicle can drive with its speed limit. Scenario 3 follows preceding vehicle successfully without been hampered by red light. both scenarios are with no time loss. Scenario 1 and 2 experienced time loss at red. It is obvious that, at the same intersection i , the link cost has following relations:

$$T^*{}_{n,i}(i_0) < T^*{}_{n,i}(i_3) \leq T^*{}_{n,i}(i_1) \leq T^*{}_{n,i}(i_2) \tag{38}$$

A Lowest-Cost-First Search is applied to obtain the minimal travel time $\min \sum_{i=1}^{I} T^*{}_{1,i}$. Apparently, the travel time reaches minimal when the idea case is that all scenarios are scenario 0:

$$\underbrace{\sum_{1}^{I} T^*{}_{n,i_0}}_{thoertically\ minimal\ value} < \sum_{1}^{I_0} T^*{}_{n,i_0} + \sum_{1}^{I_1} T^*{}_{n,i_1} + \sum_{1}^{I_2} T^*{}_{n,i_2} + \sum_{1}^{I_3} T^*{}_{n,i_3} \tag{39}$$

Note that this directed graph is not a fully connected one. Some links are disconnected due to the estimated arrival moment or the presence of a preceding vehicle. A vehicle may not drive with a scenario 0 along all the blocks. In this case, replacing one of the scenarios into another scenario with the least cost for vehicle n achieves the minimal cost that are feasible.

$$\underbrace{\sum_{1}^{I} T^*{}_{n,i_0}}_{not\ feasible} < \underbrace{\sum_{1}^{I-1} T^*{}_{n,i_0} + T^*{}_{n,i_{\neg 0}}}_{feasible} < \sum_{1}^{I_0} T^*{}_{n,i_0} + \sum_{1}^{I_1} T^*{}_{n,i_1} + \sum_{1}^{I_2} T^*{}_{n,i_2} + \sum_{1}^{I_3} T^*{}_{n,i_3} \tag{40}$$

Therefore, the Lowest-Cost-First Search leads to a greedy heuristic to try to plan scenario 0 or scenario 3 first, then scenario 1 or 2 will be considered.

2.3 SYNTHESIZED ALGORITHM

In lower-level control, the optimal control has been set up for each vehicle to calculate their optimal trajectories. In upper-level control, the scenarios are developed. In each scenario, the way to find the minimum travel time has been introduced. The problem is to minimize the total travel time for all vehicles therefore the hierarchical control is addressed systematically in a synthesized way.

According to the analysis of scenarios, scenario 0 is designed as the vehicle that can drive with its speed limit. Scenario 3 follows preceding vehicle successfully without been hampered by red light. both scenarios are with no time loss. Scenario 1 and 2 experienced time loss at red. It is obvious that, at the same intersection i , the travel time for each scenario has following relations:

$$T^*{}_{n,i}(scnario\ 0) < \ T^*{}_{n,i}(scnario\ 3) \leq T^*{}_{n,i}(scnario\ 1) \leq T^*{}_{n,i}(scnario\ 2). \qquad (41)$$

Apparently, the travel time reaches minimal when an ideal condition can happen that all scenarios are scenario 0. Nevertheless, a vehicle may not be able to drive with scenario 0 along all the blocks. In this case, replacing one of the scenarios into another scenario with the least cost for vehicle n achieves the minimal costs that are feasible. Therefore, a greedy heuristic is to try to plan scenario 0 or scenario 3 first, and then to plan scenario 1 or 2.

Define $z_{n,i} \triangleq [u_{n,i}(0)^T, \dots, u_{n,i}(t-1)^T]^T$ as the decision variable of vehicle n from the time instant 0 to t in each intersection i. Once a selection of scenarios is made, the minimal travel time ${T_{n,i}}^*$ is calculated. The decision variables of the preceding vehicle $\bar{z}_{n-1,i}$ and the constraints inputs into the next calculation. By assuming there are N vehicles and I intersections, the calculation process is listed as follows:

- Start: start with intersection $i = 1, n = 1$
- Step 1: If $0 \leq s_{n,i}(t) < s_{n-1,i}(t) \leq l_i$, go to Step 2a; Else, go to Step 3a
- Step 2a: $M_{n,i} \in [\theta_i + C * (k-1), \theta_i + G_i + C * (k-1)], k = 1$, obtain the numerical solution as scenario 0 Else, go to Step 2b
- Step 2b: $M_{n,i} \in [\theta_i + C * (k-1), \theta_i + G_i + C * (k-1)], k > 1$, obtain the numerical solution for optimal control problem as scenario 1
- Step 3a: $M_{n,i} \in [\theta_i + C * (k-1), \theta_i + G_i + C * (k-1)], k = 1$, obtain the numerical solution for optimal control problem as scenario 3 Else, go to Step 3b
- Step 3b: $M_{n,i} \in [\theta_i + C * (k-1), \theta_i + G_i + C * (k-1)], k > 1$, obtain the numerical solution for optimal control problem as scenario 2 (following a CAV) or scenario 3 (following an HV)
- Step 4: Find the solution ${z_{n,i}}^*$ for vehicle n at intersection i and broadcast all the outputs from current plan to all other CAVs. the known decision variables are $\bar{z}_{n-1,i}$ then.
- End: end by $i = I, n = N$.

As described in the algorithm, the controller determines each CAV individually and broadcasts their information and solutions. Information broadcasts to the follower if it is a CAV. This proceeds until all the vehicles have solutions for trajectory profiles. The process is demonstrated in Figure 4.

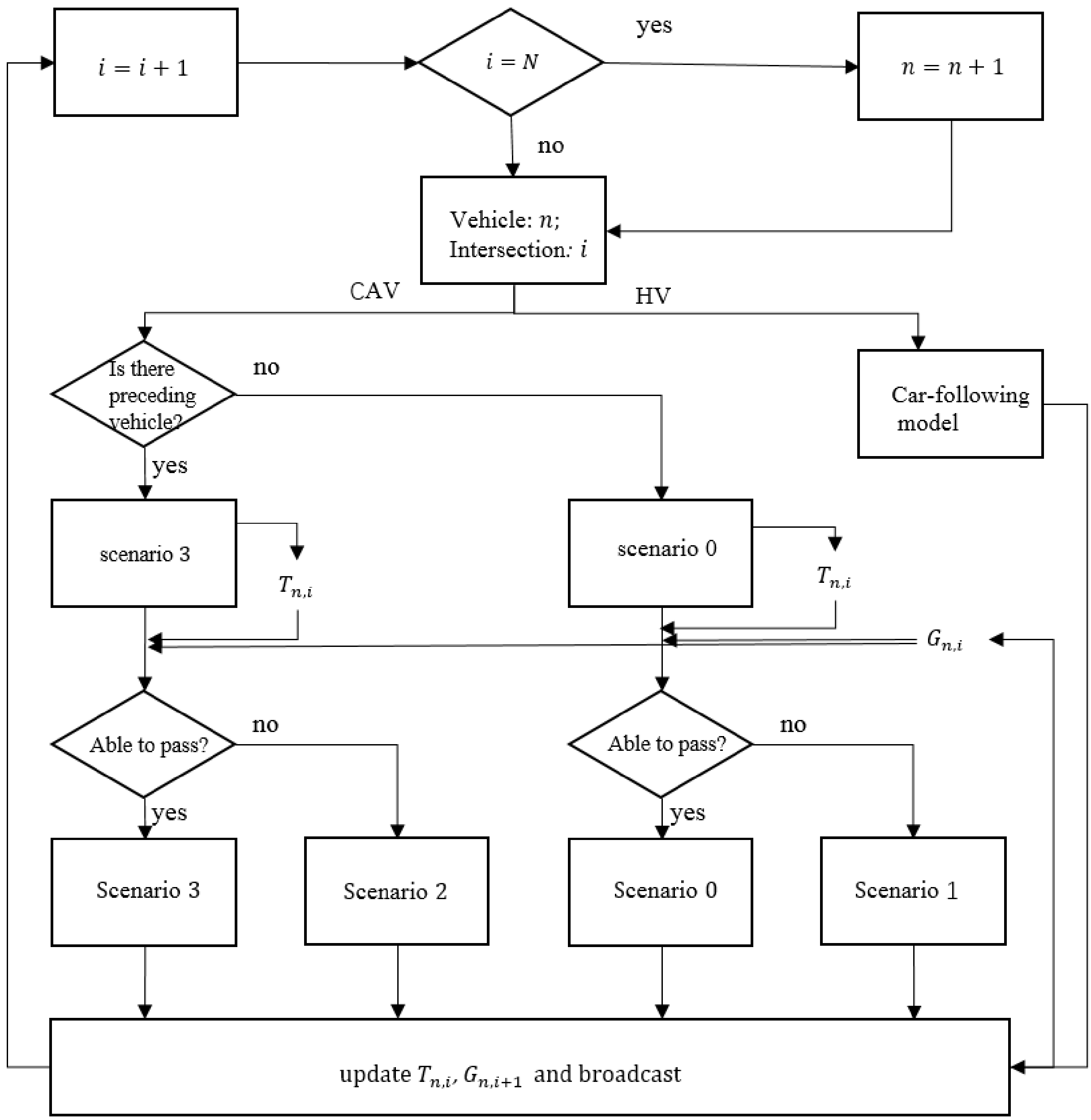

Figure 4 The flow chart of the method at the strategical level

3. EXPERIMENTS AND RESULTS

3.1 EXPERIMENT DEVELOPMENT

The proposed method was implemented in MATLAB, and numerical simulations were demonstrated below. To test under light traffic gave no value since no traffic backpropagation will happen. So only the cases when demand was moderate were considered. Two cases were presented to validate the method. Case 1 compared the method with the situation when all vehicles were HVs. HVs were assumed to slow down and stop when approaching a signalized intersection if they expected to fail to pass and remain standstill at the stop bars during the red phases. HVs were assumed to follow preceding vehicles using the intelligent driver model (IDM) model(Treiber et al., 2000). Case 2 compared the proposed method with a benchmark when all CAVs drive smoothly to avoid stop at intersections without consideration of minimal travel time.

Both cases comprised two examples. In one example, the initial average headway input was set as 5 seconds. In another example, the initial input headway was 3 seconds. The desired headway for a CAV and the IDM model was set as 3 seconds; the desired headway for a CAV following an HV was set as 4 seconds for safety concerns. Multiple runs with random seeds were applied in each case to calculate the average travel time saving under each penetration rate. The parameters used in the experiment were listed in Table 2:

Table 2 value of parameters used in the experiments.

Parameters	**Notation**	**Value**
Block length (m)	l_i	1000
Number of blocks	N	3
Number of vehicles	I	60
Cycle length (sec)	C	100
Green duration (sec)	G_i	60
Offset(sec)	θ_i	30
Weight for terminal distance	w_1	10
Weight for terminal speed	w_2	10
Number of vehicle inputs		60
Safe distance (meter)	d_s	5
Vehicle length (meter)	d_v	5
Desire headway (sec)	$t_{0,i}$	3
Desire headway a CAV following an HV (sec)	$t_{0,i(HV)}$	4
Maximal speed (m/s)	$v_{n,i,ub}$	20
Minimal speed (m/s)	$v_{n,i,lb}$	5
Maximum acceleration ($\mathrm{m/s^2}$)	$u_{n,i,ub}$	2
Minimal deacceleration ($\mathrm{m/s^2}$)	$u_{n,i,lb}$	-2

3.2 CASE 1 : PERFORMANCE UNDER DIFFERENT PENETRATION OF CAVS

Case 1 compares the results when no CAVs and when some CAVs using the proposed are applied. The simulated results were presented in Figure 5 and Figure 6.

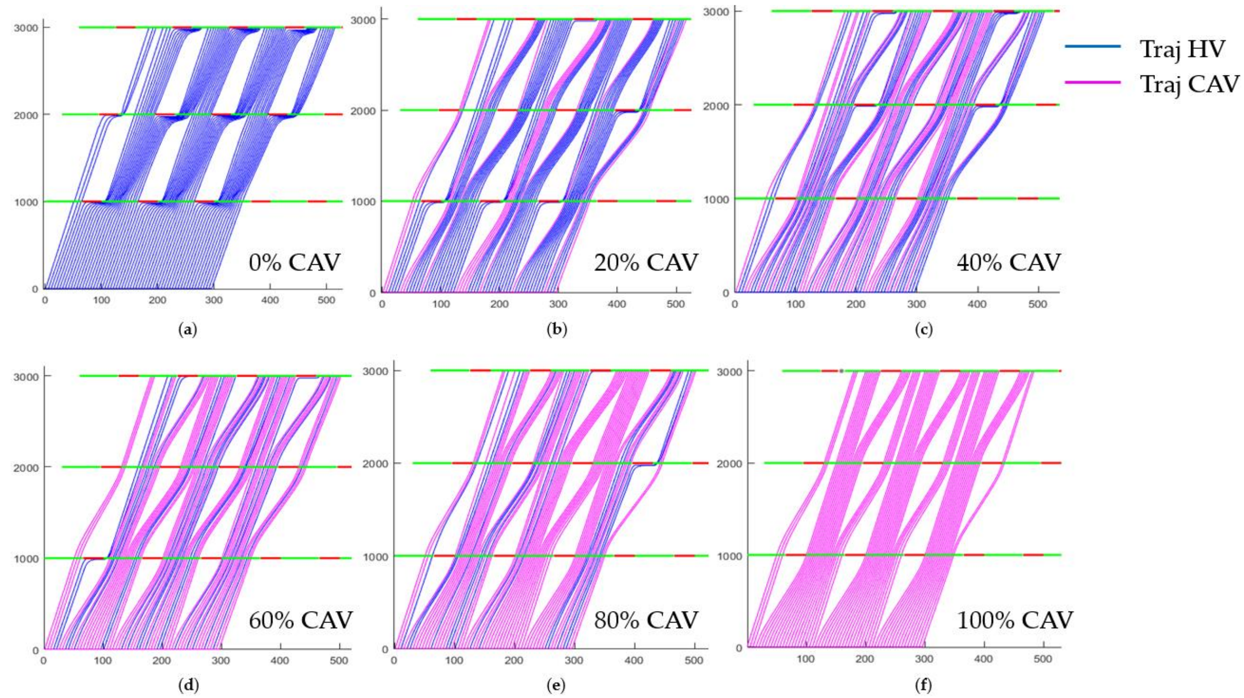

Figure 5 Trajectories of HVs (blue lines) and CAVs (magenta) under varying penetration rates of CAV when the demand was moderate (x-axis – time (sec), y -axis – distance (m))

When the initial headway for CAVs was 5 sec, the CAVs trajectories could lead the whole platoon to decompose and reconstruct reasonably. This led to a reduction in travel time in the first step. The results also showed the proposed method can reduce the number of stops; as a result, the queues and backpropagation shockwaves were mitigated to reduce the startup time, which saved travel time in the second step. The method compressed the headways for CAVs when the initial headway was larger than the desired headway, which made the traffic stream compact, leading to a reduction of travel time in the third step. Compared to the situations when all vehicles are HVs (0%), the effects of mitigation of adverse phenomena became more significant with the increase of penetration rates. When the penetration rate was 100%, the stops were mostly eliminated, and no queue and backpropagation shockwave showed.

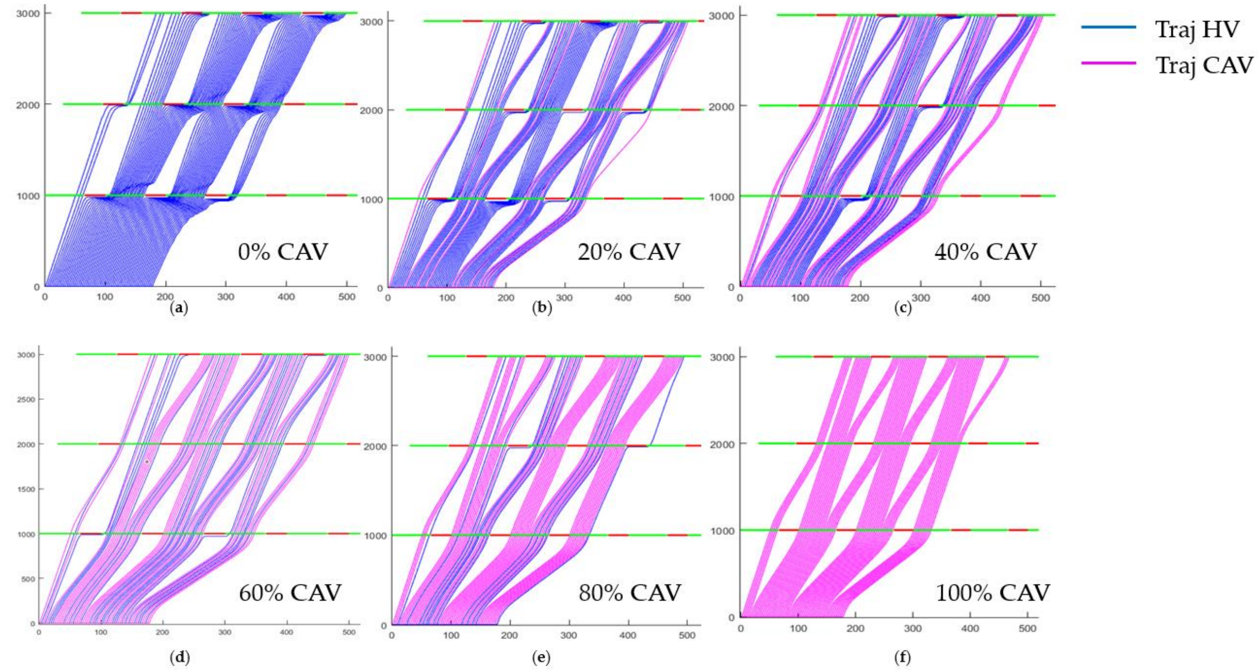

Figure 6 A comparison of trajectories between HVs (blue lines) and CAVs (magenta) under varying penetration rates of CAV, when the demand was high (x-axis – time (sec), y -axis – distance (m))

When traffic demand was higher, according to Figure 6, although the initial headways were so small that they cannot be compressed, travel time was saved from the first two steps: The whole platoon still decomposed and reconstructed in a manner to ensure vehicles to pass with the shortest time, and queues and backpropagation shockwaves were also mitigated. Overall results after multiple runs were presented in Figure 7.

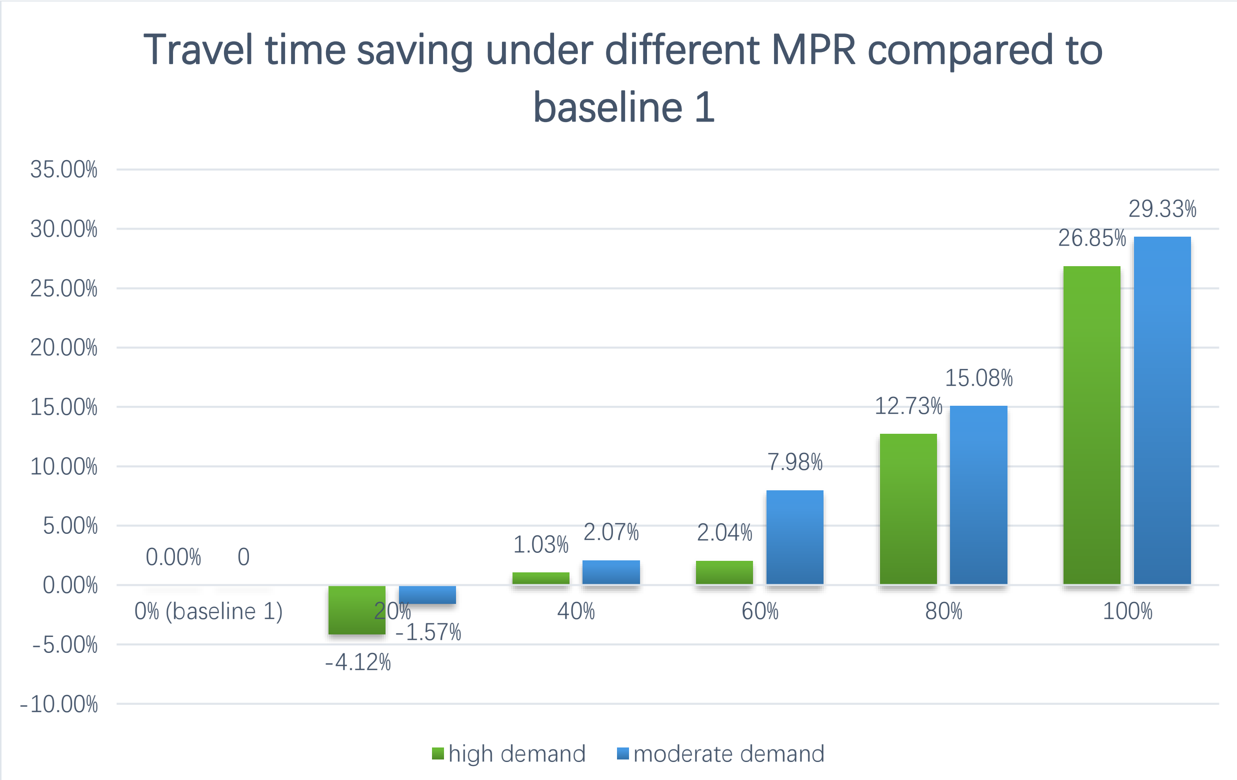

Figure 7 Travel time saving using the proposed method compared to baseline 1 under different penetration rates of CAVs.

When the penetration rate of CAVs was as low as 20%, the methods could lead to a negative effect (-1.57% and -4.12 %). The reason was that a large desire headway (4 seconds) for a CAV following an HV was set to ensure safety, which was larger than the case when a CAV followed a CAV (3 seconds), or an HV followed an HV (3 seconds). However, with the increasing penetration rates of CAV, travel time savings become effective. The travel time savings were significant when the penetration rate was larger than 60%, for both cases. When a full penetration rate was assumed, the proposed method can provide travel time saving of 29.33 % and 26.85 % in two examples.

3.3 CASE 2: COMPARE WITH A BENCHMARK

Benchmark was configured with the following settings: 1) trajectories of HVs were generated in the same way in case 1; 2) trajectories of CAVs were generated based on a benchmark. For case 2, only optimal control was used to smooth the trajectories of leading CAV at an intersection, and the others followed their leaders. Similarly, in these cases, different initial headways were demonstrated.

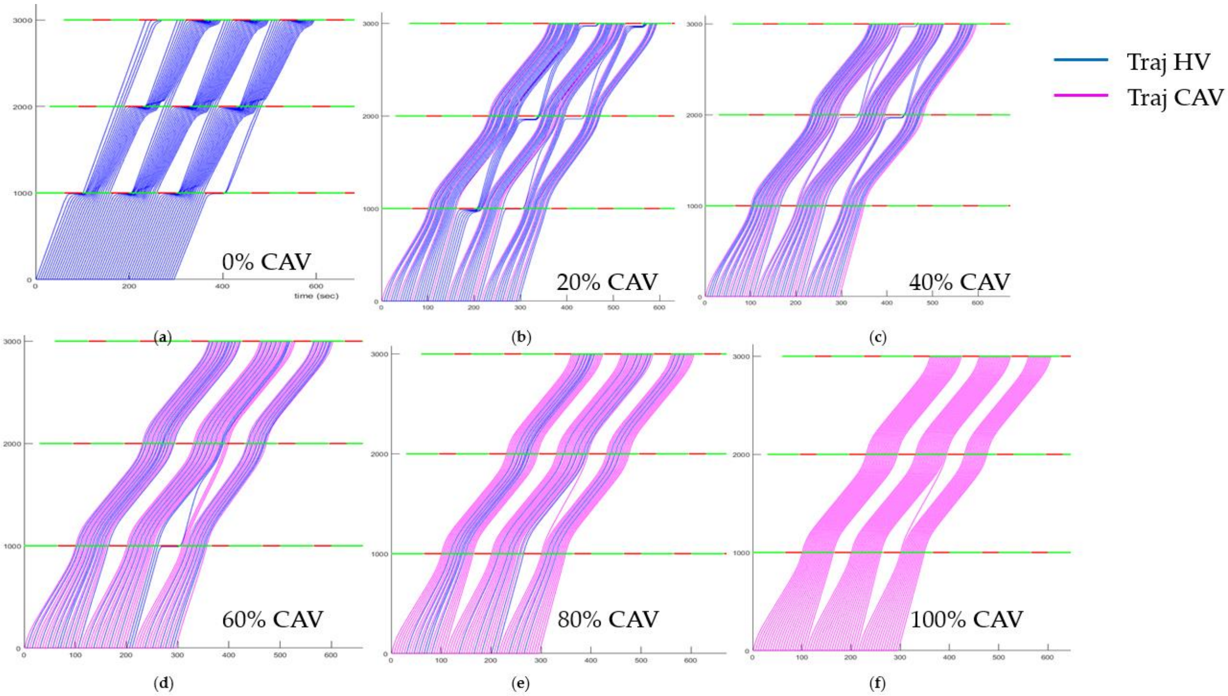

Figure 8 The trajectories between HVs (blue lines) and CAVs (magenta) under varying penetration rates of CAV when the demand was moderate (x-axis – time (sec), y -axis – distance (m)) using Baseline 1 for HVs and Baseline 2 for CAVs

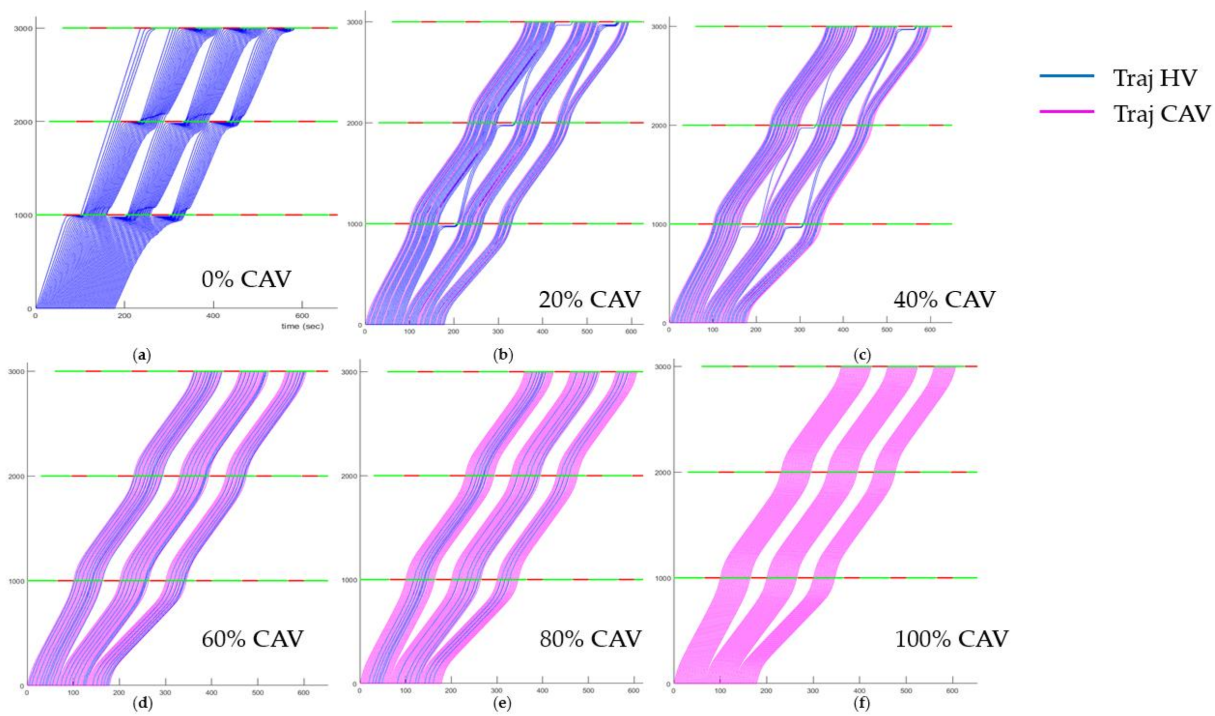

Figure 9 The trajectories between HV (blue lines) and CAV (magenta) under varying penetration rates of CAV when the demand was high (x-axis – time (sec), y -axis – distance (m)) using Baseline 1 for HVs and Baseline 2 for CAVs

As seen in Figure 8 and Figure 9, although smooth trajectories could reduce travel time by reducing time-consuming stop and startup driving behaviors at an intersection, they led to an increase in travel time if multiple intersections were involved and the local minimal travel time is not considered. This case showed the importance of the proposed method to calculate the minimal travel time locally under all possible scenarios. The outputs from case 1 and case 2 showed a significant difference in Figure 10.

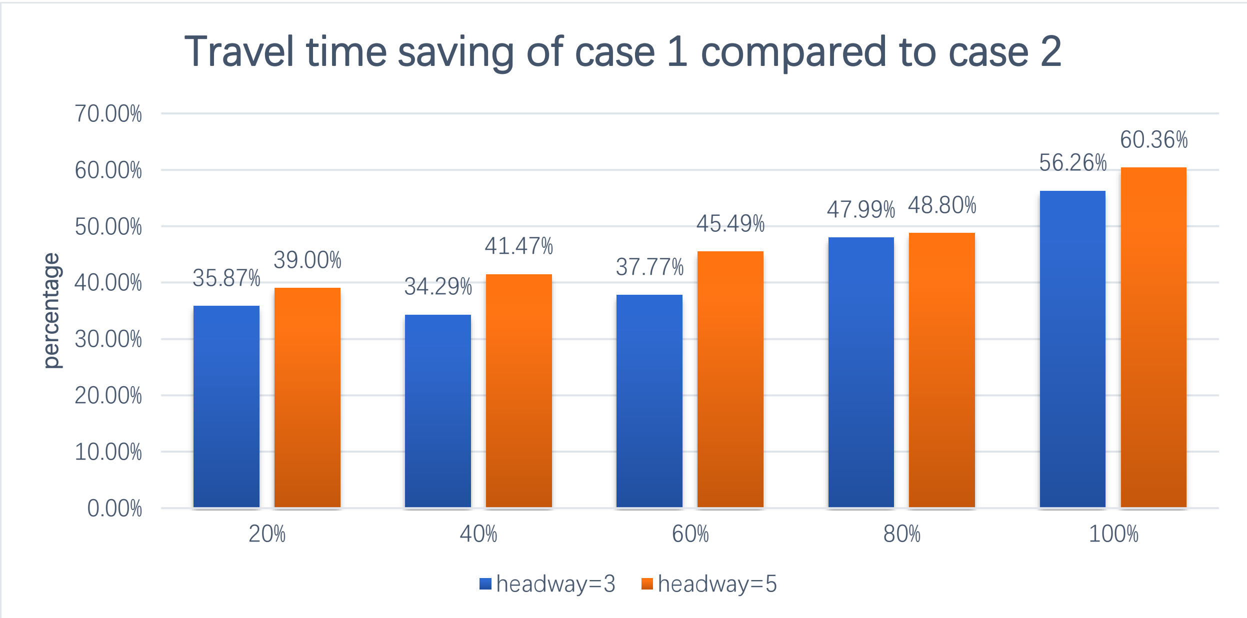

Figure 10 Travel time saving in case 1 compared to case 2 under different penetration rates of CAVs.

Comparing case 1 (using the proposed method to control CAVs) with case 2 (using a benchmark), 35.87% (shorter headway) and 39.00% (larger headway) travel time savings were shown even when the penetration rate was as low as 20%. The percentage increased to 56.26% and 60.36% when a full penetration rate was assumed.

4. CONCLUSION

On development of the lower-level layer of this hierarchical longitudinal control, mathematical formulations were developed for the relations between vehicles and signals during the time vehicles were traveling along consecutive signalized intersections. In the upper-level layer, the conditions of vehicles are decomposed into four scenarios. In each scenario, a minimal travel time is calculated. A synthesized algorithm is used to connect lower-level and upper-level layers.

Two cases were developed to validate the proposed control strategy. Case 1 concerned a non-CAV setting and Case 2 assumed all CAVs drive only with smooth trajectory without considering travel time. The proposed method significantly reduced the number of stops.
The problem was addressed at two levels: At the individual vehicle level, mathematical formulations were developed for the relations between vehicles and signals during the time vehicles were traveling along consecutive signalized intersections. We defined the control of vehicle trajectories into four scenarios. In each scenario, a constrained optimal control was developed and solved. The solutions to previous problems became the constraints and inputs for the next. Therefore, we developed an algorithm at the strategical level to allocate the vehicles in their suitable scenarios, assuming the accumulative optimal solutions yield a reduction of total travel time.

Two baselines were developed to compare with the proposed method. Baseline 1, a non-CAV setting, represented a situation when vehicles tried to travel urgently as they could. Baseline 2 was that only the state-of-the-art trajectory smooth method was applied at an isolated intersection without the consideration of local practical travel time. It represented a situation when all vehicles were not urgent so that they formed smooth traffic. The proposed method led to a situation in between and significantly reduced stops. When it came to travel time savings, compared to baseline 1, when the demand was moderate, the travel time saving ranged from -1.57% to 29.33 %. When the demand was high, the travel time saving was still significant (ranging from -4.12 % to 26.85 %). Compared to case 2 that uses baseline 2 as the control strategy of CAVs, the proposed method can save travel time from 35.87% to 56.26 % and 39.00% to 60.36%. When two types of CAVs were mixed, the performances were stable. The proposed method could reduce the travel time from 30.70 % to 52.00% considering different portions of vehicle types under different levels of traffic demand.

The method saved travel time from three aspects: First, it considered local practical travel time for each scenario, which minimizing the travel time for individuals. Second, by smoothing trajectory, stops and queues were reduced, which mitigated traffic oscillation and queue backpropagation. What is more, the method compressed the headway for a traffic stream, which saved travel time to an extent.

The contributions of the research are highlighted below:

- Formulated and decomposed scenarios of CAV longitudinal control
- Considered periodically decomposes and forms of CAV platoon to minimize travel time so that the role of a CAV could change between a follower and a leader when entering the next intersection.

The advantages of the proposed method were listed below:

- Travel time could be reduced when the penetration rate was larger than 40% and significantly reduced when the penetration rate was larger than 60%.
- The method was computational efficient. It was also extendable to consider a different number of intersections or lengths of blocks.

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