Analysis of a Straight Ladder Slippage

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ABSTRACT

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1. INTRODUCTION

"The number of emergency room visits from ladder-related injuries totaling more than 222,000 more than those for lawn mowers and home workshop saws combined," warns John Drengenberg, manager of Consumer Affairs at Underwriters Laboratories Inc."[1]. Clearly the financial loses due to these accidents are measured in hundreds of dollars per year.

There is a large number of publications that discuss the safety and the proper use of the different variety of ladders. This paper deals only with straight or extension ladders. These ladders can be mounted in two different ways as shown in Figure 1:

1. "Wall Mounting" – In this case, the top of the ladder leans against a wall, as shown in Figure 1a; and
2. "Edge Mounting" – In this case, the ladder is leaning against an edge at some point along its length as shown in Figure 1b.

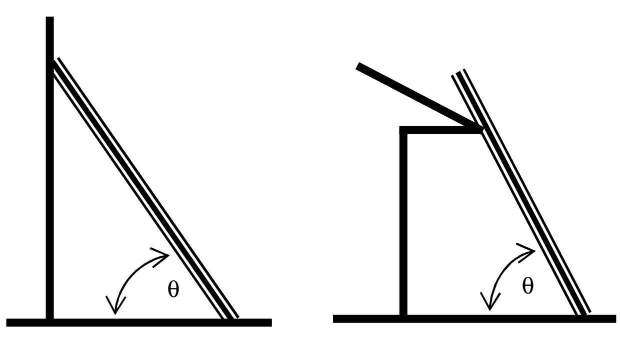

Figure 1: Straight ladder mounting.

"Wall Mounting" is very common in applications such as cleaning, painting etc. In depth analysis of this case is given in [2]. Two important conclusions are provided:

1. The static coefficient of friction at the ladder's foot should be at least 0.7104, regardless the surface and its condition, in order to prevent slip out.
2. The stability of the ladder is relatively insensitive to the coefficient of friction between the wall and the top of the ladder.

Further investigations and experimental result of the case of mounting are described in [3-6]. The major conclusions of these studies are:

1. The ladder mounting angle, θ, is the most critical parameter affecting the required friction at the ladder's feet (for ascending or descending).
2. The required coefficient of friction, at the ladder feet, and the associated slip out risk increase as the subject climbs higher.

"Edge Mounting" is very common when a person is climbing to higher surface, such as a roof. For unknown reason no analysis of this case was found. This paper will attempt to evaluate the stability conditions of a ladder that is mounted this way

2. ANALYSIS

In impending motion conditions, when the ladder just starting to slide, only one of the friction forces F_1 or F_2, shown in Figure 2, reaches its maximum available value of:

$$F = \mu N \qquad (1)$$

where N is the normal force and μ is the static coefficient of friction (COF) at the contact points. Since it is not known at which point slippage first ocurrs, the values of F_1 or F_2, cannot be assigned according to Eq. 1.

As a first step in the analysis, it is important to determine where the slippage occurred, at the bottom (point A) or at the top (Point B) contact point. Three equilibrium equations are available in equilibrium:

$$\begin{aligned}
\sum M_A &= Wa\cos\alpha - N_B L = 0 \\
\sum M_B &= N_A L\cos\alpha - F_A L\sin\alpha - W(L-a)\cos\alpha = 0 \\
\sum F_y &= N_A + N_B\cos\alpha + F_B\sin\alpha - W = 0
\end{aligned} \qquad (2)$$

where W represents the equivalent of all vertical loads, W_i, applied at a distance a_i from the bottom of the ladder:

$$a = \frac{\sum W_i a_i}{\sum W_i} > 0 \qquad (3)$$

where a is the point of application of W along the ladder as shown in Figure 1. The relationship between the two friction forces can be obtained from Eq. 2:

$$F_A = \frac{W}{2}\frac{a}{L}\sin 2\alpha - F_B \cos\alpha \qquad (4)$$

Since F_A should be positive, this result indicates that the friction force at the bottom of the ladder is always larger than the friction force at the top of the ladder. For similar COF at A and B it means that the friction force F_A will reach its maximum available value first.

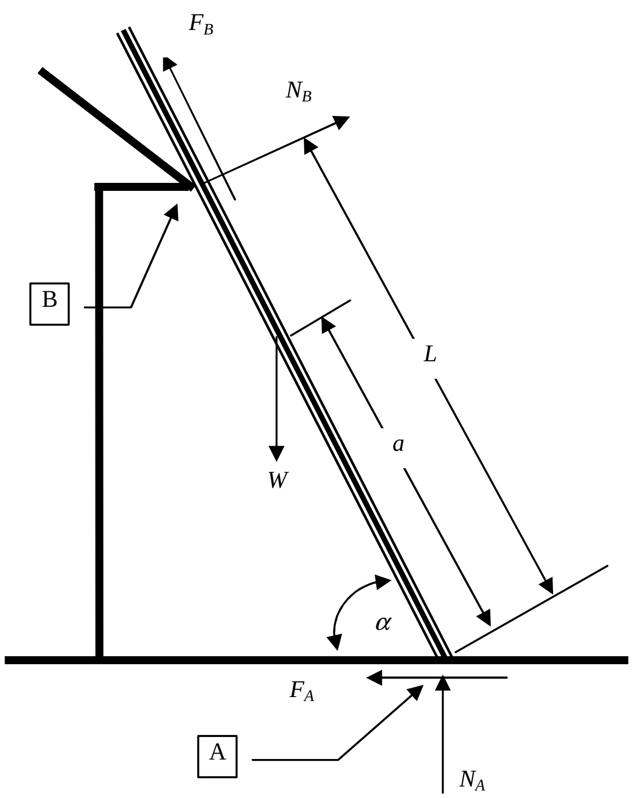

Figure 2: Free Body diagram of a ladder leaning on edge.

Case I: No friction at point B

In this case $\mu_B=0$ and therefore $F_B=0$. At impending motion, F_A reaches it maximum value of $\mu_A N_A$. Eq. 2 becomes:

$$\begin{aligned} \sum M_A &= Wa\cos\alpha - N_B L = 0 \\ \sum M_B &= N_A L\cos\alpha - \mu_A N_A L\sin\alpha - W(L-a)\cos\alpha = 0 \\ \sum F_y &= N_A + N_B\cos\alpha - W = 0 \end{aligned} \qquad (5)$$

Thus, the avoid slippage:

$$\mu_A \geq \frac{a\sin 2\alpha}{2(L - a\cos^2\alpha)} \qquad (6)$$

Defining $k=a/L$ $(0<k<1)$ yields:

$$\mu_A \geq \frac{k\sin 2\alpha}{2(1 - k\cos^2\alpha)} \qquad (7)$$

The graphs in Figure 3 illustrate the required COF for stability as function of k for different mounting angles. The following conclusions can be drawn:

1. As the loading point is closer to point B higher value of COF is required for stability (no slip out).
2. As the mounting angle increases, lower value of COF is needed for stability.
3. For $\alpha=75.52°$ (recommended by ANSI A14.2) the COF should be higher than 0.5 for $k>0.675$ which might be one rung above the mid point of the ladder span from ground to the edge support. (COF=0.5 is considered to be the threshold in Slip & Fall cases).

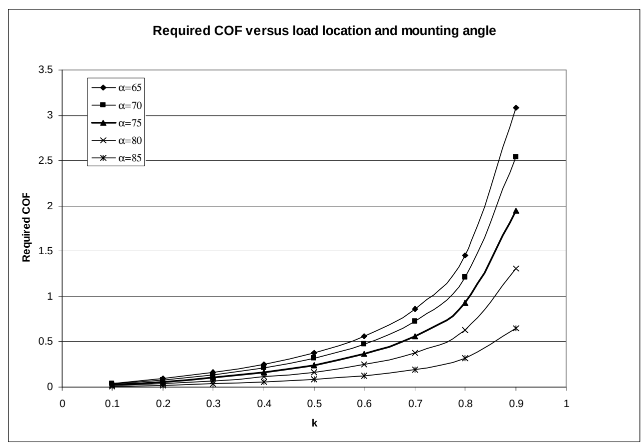

Figure 3: Required COF at point A to avoid slippage.

Case II: No friction at point A

In this case $\mu_A=0$ and therefore $F_A=0$. At impending motion, F_B reaches it maximum value of $\mu_B N_B$. Eq. 2 becomes:

$$\sum M_A = Wa\cos\alpha - N_B L = 0$$
$$\sum M_B = N_A L - W(L-a) = 0 \qquad (8)$$
$$\sum F_y = N_A + N_B\cos\alpha + \mu_B N_B \sin\alpha - W = 0$$

Thus, the avoid slippage:

$$\mu_B \geq \tan\alpha \qquad (9)$$

The result in Eq. 9 indicates the required COF at point B for mounting angles in the range of $65° < \alpha < 85°$, commonly practiced, is very high which is very high $2.144 < \mu_B < 11.43$ and cannot be achieved in most working conditions.

3. TESTING

In case of slippage it is important to determine the COF at both points of contact A and B in order to determine the cause of the accident. In the following a simple testing method, by which μ_A can be determined easily is discussed.

As shown in Figure 4, the ladder is suspended, at its top or any other known location such that $l>a$, by a vertical cable and it bottom foot is on the floor. A horizontal force, P, is applied and slowly increased until impending motion occurs.

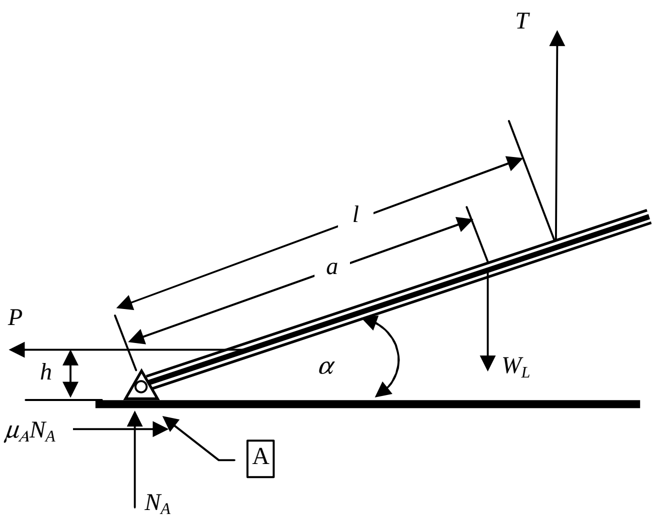

Figure 4: Free body diagram of a suspended ladder.

In this case the equilibrium equations are given by:

$$\sum F_x = \mu_A N_A - P = 0$$
$$\sum F_y = N_A - W_L + T = 0 \qquad (10)$$
$$\sum M_A = Ph - W_L a\cos\alpha + Tl\cos\alpha = 0$$

The solution of these equations is given by:

$$T = \frac{W_L a\cos\alpha - Ph}{l\cos\alpha}$$
$$N_A = W_L\left(1-\frac{a}{l}\right) - \frac{Ph}{l\cos\alpha} \qquad (11)$$
$$\mu_A = \frac{Pl\cos\alpha}{W_L(l-a)\cos\alpha - Ph}$$

In low friction conditions it might be difficult to measure the value of P accurately. One solution for that is to add a known vertical load, Q, as close to point A, as shown in Figure 5.

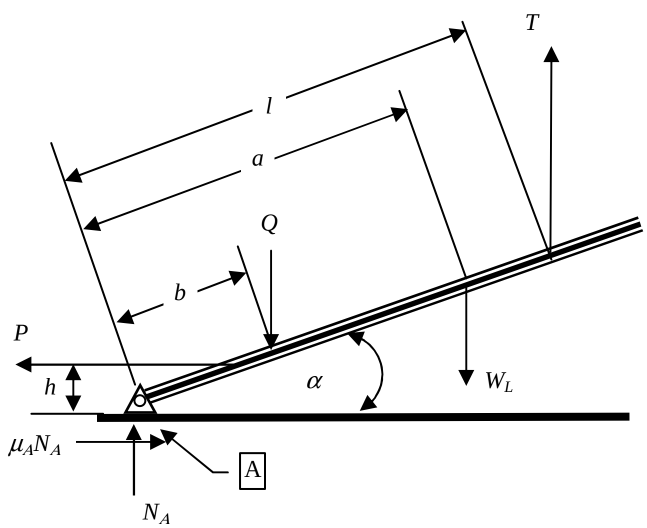

Figure 5: Additional load to the system of Figure 4.

In this case:

$$\mu_A = \frac{P(l\cos\alpha - h)}{[W_L(l-a) + Q(l-b)]\cos\alpha} \qquad (12)$$

Once the COF μ_A is found, the COF at point B can be easily determined by the following experiment. Small wheels are installed at the bottom of the ladder and the mounting angle is decreased from 90° to the point where the ladder loses its stability. At this point the angle α is recorded and Eq. 9 can be used to determine $\mu_{B.}$

4. CONCLUSIONS

The stability of a straight ladder leaning against an edge was analyzed. It is evident the friction between the ladder and the edge has small contribution toward the stability of the ladder. Therefore, it is extremely important to mount the ladder on a high friction surface. To ward that the user has to make sure that the ladder's feet are installed with rubber pads and the mounting angle is approximately 75°.

5. REFERENCES

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