Challenge questions (your work or explanation must support your answer for credit):

1. Four roommates get their cell phones mixed up and decide that each will just take one at random.

   (a) Is it more likely for everyone to get the right phone or for no one to get the right phone?

1234 2134 •3124 4123
1243 2143 3142 •4132
1324 •2314 3214 •4213
•1342 2341 •3241 4231
•1423 2413 3412 4312
1432 •2431 3421 4321

P(all right) = $\frac{1}{24}$ ○

P(none right) = $\frac{9}{24}$ □'s

   (b) Is it more likely for exactly one person to get the right phone or for exactly two people to get the right phone?

P(1 right, 3 wrong) = $\frac{8}{24}$ (•)

P(2 right, 2 wrong) = $\frac{6}{24}$ (no marks)

2. Suppose a couple has four children. Is it more likely that the gender breakdown is 2-2 or 3-1?

X = # girls, $X \sim$ binomial($n=4$, $p=.5$)

$P(X=2) = \binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = \frac{6}{16}$

$P(X=1 \text{ or } X=3) = \binom{4}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3 + \binom{4}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^1 = \frac{8}{16}$

3. Which family would be more likely to have an exact 50/50 gender breakdown, one with four children or one with 10 children? X = # girls

4 children: $X \sim bin(4, .5)$ $P(X=2) = \binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = \frac{6}{16} = .375$

10 children: $X \sim bin(10, .5)$ $P(X=5) = \binom{10}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^5 \cong .246$

4. Which would you expect to have more days on which more than 60% of its births are girls, a hospital that has 10 births per day, or a hospital that has 50 births per day?

10 births/day: X = # girls $\sim bin(10, .5)$

$P(X > 6) = P(X = 7 \text{ or } 8 \text{ or } 9 \text{ or } 10) = .172$

50 births/day: X = # girls $\sim bin(50, .5) \approx N(25, \sqrt{12.5})$

exact: $P(X > 30) = 1 - P(X \leq 29) = 1 - .941 = .059$

approx. (normal): $P(X > 30) \cong P\left(Z > \frac{30-25}{\sqrt{12.5}}\right) = P(Z > 1.41) \approx .079$