# Honors Introduction to Statistics
# Lab 8: Means – Round 1

A study published in the *Journal of the American Medical Association (1992)* called into question the traditional value of 98.6 degrees Fahrenheit as the typical healthy body temperature. Subjects were 130 healthy men and women, aged 18-40 years, who were volunteers participating in *Shigella* vaccine trials at the University of Maryland Center for Vaccine Development, Baltimore. The researchers took the subjects' normal oral temperatures at several different times during three consecutive days using a digital thermometer. The mean temperature for each subject was recorded, along with gender. The data are located in the temps.ftm file in the Instructor Materials / Dr. Fitchett / STA 2023 folder.

1. Open the file and produce graphical and numerical summaries to describe the distribution of all body temperatures in the sample. Summarize what they reveal (describe the shape, center, and spread of the distribution, as well as any deviations from the overall pattern; report summary statistics).

   The distribution of temperatures is roughly symmetric & single-peaked w/ a center (mean) of 98.25°F, a std dev of .733°F, a min of 96.3 & max of 100.8°F. There are potential (mild) outliers on both the high and low end.

2. Does it appear that the sample mean is near 98.6 degrees? No, it's about 98.25 which is close, but not super close.

3. Do you think the distribution of temperatures could be modeled by a normal distribution? Explain why or why not.

   Yes, with the exception of the one high outlier, the normal quantile plot is quite linear.

4. Right now, we are assuming three (unrealistic) things for computing confidence intervals and conducting hypothesis tests for means. List the three assumptions and comment on the degree to which each assumption is met in this example.

   • SRS (not met, but sample may be representative)

   • $X \sim N(\mu, \sigma)$ seems OK

   • $\mu$ unknown, $\sigma$ known. We don't know either $\mu$ or $\sigma$.

**For the computations we do below, we will assume that the standard deviation for the temperature of all healthy men and women, aged 18-40 years is σ = 0.75 degrees Fahrenheit.**

5. Describe carefully each of the following items in this study with words, symbols and units:

   - Population healthy Amer adults 18-40
   - Population parameter of interest avg temp of all healthy adults 18-40, $\mu$, °F
   - Sample (include its size) temps of the 130 participants
   - Sample statistic that estimates parameter of interest

     Sample mean temp of 130 participants, $\bar{x} = 98.25$ °F

**Confidence Intervals**

6. Calculate, by hand, a 95% confidence interval for your parameter of interest.

$$\bar{X} \pm z^* \frac{\sigma}{\sqrt{n}}$$

$$98.249 \pm 1.96\left(\frac{.75}{\sqrt{130}}\right)$$

$$98.249 \pm .129$$

7. Provide a one sentence interpretation of your confidence interval.

We are 95% confident that the true average temperature for healthy adults aged 18-40 years is between 98.12 and 98.38°F

8. Does your interval capture 98.6 degrees? What do you conclude (and with what confidence)?

No. We are pretty certain (95%) that 98.6° is not the true avg temp for this population.

9. Fathom does not make the unreasonable assumption that σ is known, but instead uses the sample standard deviation *s* as an approximation for the population standard deviation σ, and then accounts for the uncertainty in estimating σ by *s*. We will learn how to account for this additional uncertainty soon. For now, we will have Fathom compute a 95% confidence interval to see how it compares with the one we found by hand.

- Drag a new Estimate box onto the workspace. Click in the upper right hand corner of the box and choose **Estimate Mean.**
- Drag the temp attribute to the line that says "Attribute (numeric): <unassigned>" and drop it there. Note that Fathom reports the number of observations, the sample mean and standard deviation, and also something called the standard error. The standard error is our estimate of $\sigma/\sqrt{n}$, namely $s/\sqrt{n}$.
- Record the confidence interval reported by Fathom. Also write Fathom's interpretation of the confidence interval.

$98.249 \pm .127$ or $(98.122, 98.377)$

If the sampling process were performed repeatedly, the CIs generated would capture the pop mean 95% of the time

10. Does Fathom's confidence interval capture 98.6 degrees? What does this suggest about the plausibility of 98.6 as the average temperature of the population?

No. It suggests that 98.6°F is not a very plausible value for the avg temp of all healthy adults, 18-40.

**Hypothesis tests**

11. Next, we'll perform a hypothesis test to determine if the data provide evidence that the population mean temperature is different from 98.6 degrees. Write down the null and alternative hypotheses for such a test, both in words and in symbols.

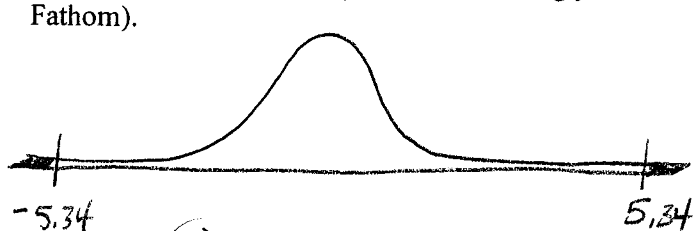
$H_0: \mu = 98.6$ the avg temp of all healthy 18-40 yr olds is 98.6°F

$H_a: \mu \neq 98.6$ ——— is not 98.6°F

12. Compute the test statistic.

$$z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} = \frac{98.249-98.6}{.75/\sqrt{130}} = -5.34$$

13. Sketch and label a normal distribution, shading the region on your graph that represents the p-value for your test. Find and record that p-value, describing your method (the standard normal table, your calculator, Fathom).




$p \approx 0$

14. Would you reject or fail to reject the null hypothesis at the $\alpha = .05$ level?

15. State your conclusion in practical terms, using a complete sentence.

The data provide very strong evidence that the avg body temp of healthy 18-40 yr olds is different from 98.6°F

16. As before, we'll next have Fathom do the work, but without the assumption of a known population standard deviation.

- Drag a new Test box onto the workspace. Click in the upper right hand corner of the box and choose **Test Mean**.
- Drag the temp attribute to the line that says "Attribute (numeric): <unassigned>" and drop it there. Note that the same summary statistics that were shown for the confidence interval are reported again. Where the box indicates "Alternative hypothesis: The population mean of **temps is not equal to** 0", leave the "**is not equal to**" alone, since we are performing a 2-sided test. Change the 0 to 98.6, and click OK.
- Fathom reports a t test statistic instead of a z test statistic. This is because Fathom is not assuming the population standard deviation is known, but is instead estimating it using the sample standard deviation. Still, the t-statistic should be similar to the z-statistic you computed above. Record the t-statistic, the p-value, and Fathom's interpretation of the test result.

$t = -5.455$, $p < .0001$

If it were true that the avg body temp for all healthy 18-40 yr olds were 98.6°F & the sampling process were performed repeatedly, the prob of getting a t-statistic w/ $|t| \geq 5.455$ would be $< .0001$

17. Following the procedure above, use Fathom to determine if the data provide evidence that the population mean temperature is different from 98.3 degrees. Again, write the hypotheses, record the test statistic and p-value, and provide an interpretation (either Fathom's or your own).

$H_0: \mu = 98.3$

$H_a: \mu \neq 98.3$

$t = -.7895$

$p = .43$

There is no evidence that the avg body temp of healthy 18-40 yr olds is different from 98.3°F

**Duality between confidence intervals and two-sided hypothesis tests**

You may have noticed (or recalled from the class discussion) the analogy between two-sided testing and confidence intervals at the same level of confidence. Let's take another look at the analogy.

(98.12, 98.38)

18. You now suspect that 98.4 is the "correct" body temperature. Does it fall in your 95% CI?

NO

19. Using Fathom, test your new hypothesis at $\alpha = 0.05$ significance level. State your test result.

$p = .021$ (Reject at $\alpha = .05$) There is good evidence that the true avg body temp is different from 98.4°F

20. Using Fathom, compute a 99% CI for the average body temperature. Does it capture 98.4 now?

(98.08, 98.42) Yes

21. Using Fathom again, test the 98.4 hypothesis at the $\alpha = 0.01$ significance level. State the result.

$p = .021$ (same test as above) (Fail to reject at $\alpha = .01$) There is not enough evidence to reject 98.4 as the true avg body temp at the $\alpha = .01$ level.

22. Your friend did a similar hypothesis test last week (on the same data but with a different "correct" temperature $T$) and rejected the null hypothesis in favor of a two-sided alternative at the significance level 2%, but cannot remember the "correct" temperature $T$ in the rejected null hypothesis. You compute a 98% confidence interval. Does your interval capture your friend's guess $T$? Explain.

No. Rejecting the $H_0$ value at 2% sign. level corresponds to value being outside the 98% CI

23. Inspired by your results above, formulate a general rule for confidence intervals at the confidence level C and two-sided hypothesis testing at the significance level $\alpha = 1 - C$. Fill in the blanks:

> At the significance level $\alpha = 1 - C$, a test ___rejects___ the null hypothesis $\mu = \mu_0$ (in favor of a two-sided alternative) exactly when the value $\mu_0$ is ___outside___ the C confidence interval.

**What about gender?**

24. Produce a split histogram from which you can compare the distributions of temperatures for men and women in the sample. Produce side-by-side boxplots. Use a summary table to find summary statistics for the two groups. Describe what the plots reveal and record appropriate summary statistics.

Plots reveal that men tend to have slightly lower body temps than women. Also men's temps are less spread out than women's.

| | M | F |
|---|---|---|
| $\bar{x}$ | 98.10 | 98.39 |
| $s$ | .69 | .74 |

25. Do you think the mean body temperature for healthy adult men is significantly different from the mean body temperature for healthy adult women? Explain why you answered the way you did. We'll investigate this question statistically in a few weeks. For now, just use your best judgment.

There's a lot of overlap in the distributions, so difference may not be significant

26. Explain in simple words the meaning of the phrase *the difference is statically significant*.

"The difference is statistically significant" means the difference is unlikely to have occurred by chance alone.