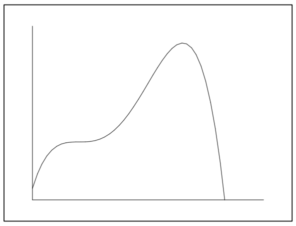
Practice for Test 3

1. (a) dy/dt gives the rate at which the water level is changing, in feet per hour, as a function of t.
   (b) Looking at a graph of the original function, we know its derivative is 0 at the points where its tangent line is horizontal. The tangent line is horizontal at the high tide and low tide times: $t = 0$, 6, 12, 18 and 24 hours after midnight.

2. Increasing on $(-\infty, 1)$, concave down on $(-\infty, 2)$

3. (a) $-17/25$ (b) $-1/4$ (c) 3 (d) 4π

4. 

5. $y - 1 = \frac{1}{9}(x - 3)$

6. (a) $\sqrt{1+x} \approx 1 + 0.5x$ for x near 0.
   (b) $\sqrt{1.1} \approx 1.05$; the approximation is larger than the value of $\sqrt{1.1}$ because the graph of $y = \sqrt{1+x}$ is concave down (so the tangent line is above the graph).

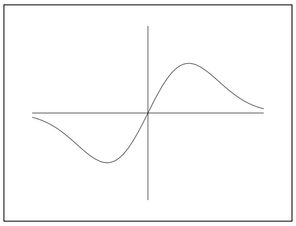
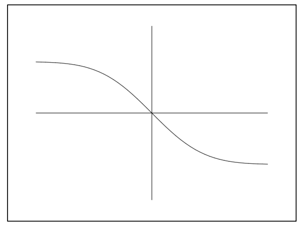
7. (a) $\sin 2$. L'Hopital's rule does not apply (the limit does not have an indeterminate form). The limit can be evaluated by substitution.
   (b) 0 (Use L'Hopital's rule and then do some algebra to simplify before substituting $x = 0$.)

8. Start with the identity $\sin(\arcsin x) = x$. Take the derivative of both sides to get $\cos(\arcsin x)\frac{d}{dx}(\arcsin x) = 1$, then solve for $\frac{d}{dx}(\arcsin x)$ to get $\frac{d}{dx}(\arcsin x) = \frac{1}{\cos(\arcsin x)}$. Use a right triangle with angle $\theta = \arcsin x$, opposite side x and hypotenuse 1 to determine that $\cos\theta = \sqrt{1-x^2}$. Plug this back in to get $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$.

9. (a) Draw a right triangle with hypotenuse 13, vertical leg $y = 5$, and horizontal leg $x = 12$. We know $x^2 + y^2 = 169$, so we can take the derivative of both sides with respect to t, then substitute the things we know and solve for $\frac{dy}{dt}$, which turns out to be -24 ft/sec.
   (b) The area of the triangle is $A = .5xy$. Taking the derivative of both sides with respect to t (and using the chain rule), gives $\frac{dA}{dt} = .5\frac{dx}{dt}y + .5x\frac{dy}{dt}$. Substituting what we know, and our result from part (a) gives $\frac{dA}{dt} = -119$ ft/sec.

10. (a) $-2, 1, 4$
    (b) max at $x = -2$, neither at $x = 1$, min at $x = 4$;
    (c) $x \approx -.5, 1, 3$. These are the places that f' changes from increasing to decreasing or decreasing to increasing (so f'' changes sign, and f changes concavity).

11. $S = kwh^2$ and $60^2 = w^2 + h^2$. Solving the second equation for h and substituting into the first gives $S = kw(3600 - w^2) = 3600kw - kw^3$. Taking the derivative with respect to w allows us to find the only critical point for the function S, namely $w = \sqrt{1200}$. Use intervals to verify that S has a maximum (not a minimum!) when $w = \sqrt{1200}$ and $h = \sqrt{2400}$ cm.

12. (a)  (c) 

(b) The function g is always decreasing (because its derivative is always negative). The graph of g changes from concave down to concave up at $x = 0$ because g' changes from decreasing to increasing at $x = 0$.

13. The profit function is given by Profit = Revenue - Cost, or $\pi(q) = 50q - C(q)$. Find the critical points for the profit function by taking the derivative, setting it equal to 0, and solving for q. The two critical points are $q = 2$ and $q = 48$. Checking intervals shows that π has a minimum at $q = 2$ and a maximum at $q = 48$, so Huey should build 48 boards per month. If he makes and sells 48 boards per month, his profit will be $\pi(48) = \$1151.52$ per month.

14. $\sqrt[3]{2V}$ by $\sqrt[3]{2V}$ by $\sqrt[3]{V/4}$