Review for Test 1

1. The number of hours of daylight varies by season and by longitude. In West Palm Beach, for instance, the number of daylight hours varies between approximately 10.5 hours (December 22), and 13.8 hours (June 21). Find a formula for the number of daylight hours in West Palm Beach as a function of time. Clearly define your independent variable.

2. The value V of a 1995 Sebring, in thousands of dollars, is a function of the age a of the car, in years. Let $V = f(a)$ be the function that represents the value of the Sebring when it is a years old.

   (a) Interpret the statement $f(5) = 8$ in practical terms. (Say something about the value of the car and its age. Include units.)

   (b) If the car was purchased for $20,000 in 1995, and V is a linear function of a, find a formula for V and sketch a graph of V against a. (Carefully label your axes.)

   (c) Explain the significance of the horizontal and vertical intercepts in terms of the value of the car.

   (d) Explain how we know that f is invertible.

   (e) Find a formula for f^{-1}.

   (f) Find $f^{-1}(5)$ and interpret your answer in practical terms.

3. Niki invested her savings in the stock market. The investment was a winner, increasing in value to $12,500 after two years, and to $15,000 after another two years. Assume the value V of Niki's investment is given by

$$V = f(t) = V_0 a^t = V_0(1+r)^t = V_0 e^{kt},$$

   where t is the number of years the money has been invested.

   (a) Find the base a.

   (b) Find the annual percentage growth rate r.

   (c) Find the *continuous* annual growth rate k.

   (d) If the trend in Niki's investment continues, when will its value be $20,000? (Be sure your answer is clear and includes units.)

4. (a) Write an equation for a graph obtained by reflecting the graph of $y = x^2$ across the x-axis, followed by a vertical upward shift of 1 unit. Sketch the graph.

   (b) What is the equation if the order of the transformations (reflecting and shifting) in part (a) is interchanged?

   (c) Are the two graphs the same? Explain the effect of reversing the order of transformations.

5. The Bay of Fundy in Canada has the largest tides in the world. The difference between low and high water levels is 15 meters (nearly 50 feet). The time between successive high tides is 12.4 hours. At a particular point the depth y, of the water, in meters, is given as a function of time t, in hours since midnight, by

$$y = D + A\cos(B(t - C)).$$

   (a) What is the physical meaning of D?

   (b) What is the value of A?

   (c) What is the value of B?

   (d) What is the physical meaning of C?

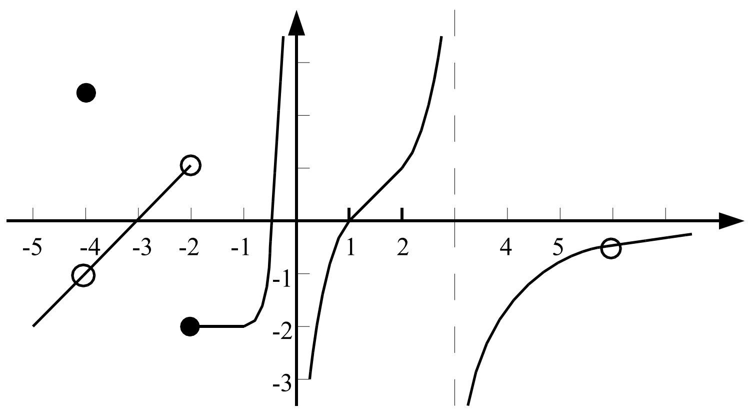
6. Values of three functions are given in the table below. Find a formula for each of the functions. Your work must support your answer. Using built-in calculator functions to find formulas will earn you at most 1/4 of the possible credit. [Hint: Each function is linear, exponential, quadratic, or cubic.]

x	$f(x)$	$g(x)$	$h(x)$
4.0	4.80	6.00	3.20
5.4	8.75	8.66	7.87
6.8	13.87	12.50	15.72
8.2	20.17	18.05	27.57

Which function is largest as $x \to \infty$? Why?

7. Write a possible formula for a rational function with exactly one horizontal asymptote at $y = 2$, two vertical asymptotes at $x = -1$ and $x = 3$, and one x-intercept at $x = 1$. Explain how you arrive at your answer. (A correct answer with no explanation will receive no credit.)

8. The graph of f is shown below.



(a) At which values of x is $f(x)$ not continuous?

(b) At which values of c does $\lim_{x\to c} f(x)$ not exist?

(c) Find the requested limits:

$\lim_{x\to -2^-} f(x)$ $\qquad$ $\lim_{x\to 3^+} f(x)$ $\qquad$ $\lim_{x\to\infty} f(x)$

(d) Suppose g is the function determined by restricting the domain of f to $0 < x < 3$. (This means g is just the portion of the graph that lies between the y-axis and the vertical asymptote at $x = 3$.) Estimate $g^{-1}(2)$.

9. Evaluate the limit algebraically. Math grammar counts!!!: $\lim_{h\to 0} \dfrac{-5(1+h)^2+5}{h}$

10. An orange is thrown into the air. Its height at various times is shown in the table.

t (seconds)	0	0.5	1	1.5	2	2.5	3	3.5
$s(t)$ (feet)	3	26.5	42.0	49.5	49.0	40.5	24.0	0

(a) Find the average velocity of the orange between $t = 2$ and $t = 3$ seconds.

(b) Use the table below to estimate the instantaneous velocity of the orange exactly one second after it is released.

t (seconds)	0	0.9	0.99	0.999	1.0	1.001	1.01	1.1	2
$s(t)$ (feet)	3	39.540	41.756	41.977	42.00	42.023	42.228	44.140	49.0

(c) Explain (and show!) how your results above (both of them) can be visualized on a graph of $y = s(t)$.