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The fractal geometry of ancient Maya settlement

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Abstract

Ancient Maya settlement patterns exhibit fractal geometry both within communities and across regions. Fractals are self-similar sets of fractional dimension. In this paper, we show how Maya settlement patterns are logically and statistically self-similar. We demonstrate how to measure the fractal dimensions (or Hausdorff–Besicovitch dimensions) of several data sets. We describe nonlinear dynamical processes, such as chaotic and self-organized critical systems, that generate fractal patterns. As an illustration, we show that the fractal dimensions calculated for some Maya settlement patterns are similar to those produced by warfare, supporting recent claims that warfare is a significant factor in Maya settlement patterning.

Keywords: Maya archaeology; Settlement patterns; Fractal geometry; Rank–size rule; Chaos theory; Self-organized criticality

1. Introduction

Human settlement patterns are typically highly complex and exhibit variation at many scales. Archaeologists have had only limited success describing, modeling, and predicting ancient settlement patterns. Part of the weakness in the archaeological approach to settlement stems from a failure to appreciate and apply models of settlement from modern geography. Modern geographers have found nonlinear science to be a fertile source of spatial models of settlement. This article focuses on the fractal analysis of Maya settlement, which will serve as an example of nonlinear description and analysis of ancient settlement.

The ancient Maya lived in the Mexican states of Campeche, Quintana Roo, Yucatán, and the eastern parts of Chiapas and Tabasco; all of Guatemala and Belize; and the western parts of Honduras and El Salvador (Fig. 1). From obscure origins in the early first millennium B.C.E., the Maya developed a powerful and refined civilization that persisted in its aboriginal form until the Spanish Conquest. Maya settlement is distinctive, although it exhibits both regional and temporal variation.

Ancient Maya settlement is fractal in various ways. It is fractal at both the intra-site and the regional levels. Within archaeological sites, settlement is fractal because the buildings form a pattern of repeated, complex, nested clusters of clusters. The spatial organization of the buildings is logically and geometrically self-similar and mathematically of fractional dimension. One can infer that the self-similarity of the pattern is structurally related to the kinship and social systems that generated it. At the regional level, settlement is fractal in several ways: (1) the size–frequency distribution of settlements is fractal; (2) the rank–size relation among sites is fractal; and (3) the geographical clustering of sites is fractal. The regional fractality of settlement may be attributable to warfare or to the economic and political factors that drive the development of central place hierarchies and lattices. To understand what these facts mean and what they imply for our vision of prehistoric Maya society, one must understand what fractals are and how they form.

2. Fractals

Fractals are self-similar sets of fractional dimension. A pattern is self-similar if it is composed of smaller-scale

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Fig. 1. Map of the Maya area showing location of major archaeological sites and cultural subregions. Drawn by Lynn A. Berg. (Subregions based generally on [46, p. 21].)

copies of itself. Here, the term "similar" carries the mathematical denotation of objects that have the same shape but differ in size. One should envision an infinite regression of smaller and smaller images that constitute a whole that is similar to its parts. Think of a fern: it is composed of branches that look like little ferns; those branches in turn are made of smaller but structurally identical elements. Because of self-similarity, fractals are also "scale invariant." Scale invariance means that fractals appear (mathematically, if not visually) to be the same at all scales of observation. Why does one have to include a scale in a photograph of a rock? Rocks appear the same at all scales of observation. Looking at a photograph, the observer cannot know what the scale really is unless there is an object of known size in the picture. This phenomenon occurs, of course, because rocks are natural fractals.

The second part of our definition says that fractals must have "fractional dimension," by which we mean that when it is measured the fractal dimension should be a fraction, not an integer. For a thing to be fractal, therefore, it is not enough for it to be self-similar. The self-similar elements must also be related in scale by a non-integral power-law. The fractal dimension of the pattern measures the power-law. The familiar concept of Euclidean dimension only includes integers: 0 for a point, 1 for a line, 2 for a plane, and so forth. Modern mathematicians have developed a number of other ways of measuring dimensions that are fractional, and therefore, strictly speaking, non-Euclidean. These methods include the correlation dimension, the information dimension, the capacity dimension, and others, all of which are mathematically related. Here we will refer to the "fractal" or "Hausdorff–Besicovitch" dimension.

The fundamental parameter of a fractal set is its fractal dimension. The dimension is described by the following relation:

$$a = \frac{1}{s^D} \tag{1}$$

where a is the number of self-similar "pieces", s is the linear scaling factor of the pieces to the whole, and D is the dimension to be calculated. D can be calculated as:

$$D = -\left(\frac{\log a}{\log s}\right) \tag{2}$$

For fractals, by definition, D is not an integer. D measures the complexity of the set and expresses the power law that relates the self-similar parts to the whole. For example, a fractal curve will have a fractal dimension between 1 (the Euclidean dimension for a line) and 2 (the Euclidean dimension for a plane). The more complex the curve, the closer the dimension will be to 2. The theoretical maximum dimension for a fractal curve is 2, when it becomes so complex it fills the plane. The dimension of a fractal object captures important information about the character of the phenomenon. The fractal dimension is also a guide to the type of nonlinear process that generated the pattern. As we examine empirical fractal patterns, we will consider the nonlinear processes that generate patterns with comparable fractal dimensions. Most of this paper involves estimating the dimensions of empirical cultural fractal patterns.

Any kind of set can be a fractal: points, lines, surfaces, multi-dimensional data, or time series. In this paper, we will be concerned with (1) fractal frequency distributions and (2) fractal geometric patterns embedded in two dimensions. Mandelbrot [63, pp. 341–348] developed the concept of the fractal distribution. Generally, a fractal distribution is one that exhibits a power-law relation, because that is the only kind of statistical distribution lacking a characteristic or inherent scale [93, pp. 1–2]. Therefore, they are scale invariant and self-similar, which are diagnostic qualities of fractals. Conceptually, the self-similarity occurs in the frequencies of sizes or magnitudes: smaller pieces are exponentially more common than large ones. A large variety of natural and sociological phenomena exhibit fractal distributions [81, pp. 103–120].

Natural fractal distributions include the number–area relation for islands [62], the size–order relation for streams [93, pp. 183–187], the frequency–magnitude relation for earthquakes [93, pp. 57–62], the size (i.e., thickness)–frequency relation for geological strata [8, pp. 78–80], the size–frequency relation for rock fragments [92], and the size–frequency distribution for biological taxa [22,23]. Cultural fractal distributions include Zipf's law of word frequencies [63, pp. 344–347], Pareto's distribution of incomes [63, pp. 347–348], the rank–size distribution of settlements [14, pp. 48–55; 42], the size–frequency relation for archaeological debitage [20], the size–frequency relation for wars [76], and the size–frequency distribution of US corporations [7]. Many of these phenomena have characteristic fractal dimensions that serve as a guide to the types of nonlinear processes that created them.

Mandelbrot [61] also developed the idea of the fractal curve. Complex curves like contour lines, coastlines, river plan views, and the shapes of mountains, clouds, lakes, and trees are all fractals, as are the outlines of lithic tools [52]. Fractal curves are ubiquitous in nature.

Many textbooks and popular works on fractals are available (e.g., [14,47,56,63,74,81,93]). We refer the readers to these works for a longer and more detailed treatment of fractals and the methods of fractal analysis.

The determination that an object is a fractal and the measurement of the fractal dimension are, in themselves, essentially descriptive exercises. Fractals are patterns (or, technically, "sets"), and so the first step in understanding them is describing them accurately and precisely. While the accurate description of these complicated patterns is not trivial, we do, nevertheless, think of description as a prelude to further explanation. We expect explanation to reveal the underlying processes that lead to pattern formation. It is, of course, a dictum of modern archaeology that the archaeological record is the static picture of past cultural dynamics (e.g., [17]). We wish to use the static fractal pattern to infer the underlying dynamical historical process. Therefore, we want to know what kinds of processes create fractals.

Dynamical processes of various kinds can generate fractal patterns. Iterated function systems, cellular automata, and diffusion-limited aggregation, for example, all produce fractal patterns and can be used to simulate known fractals that are widely observed in nature and culture [74,101]. Two major classes of nonlinear dynamic systems are well known for generating fractal patterns: chaotic systems and self-organized critical systems.

Whereas a fractal is a set, chaos is a characteristic of deterministic dynamic systems. Chaotic systems are common, perhaps more common than stable, non-chaotic ones. A deterministic dynamic system is said to be chaotic if "solutions that have initial conditions that are infinitesimally close diverge exponentially" [93, p. 219]. This fundamental characteristic of chaos is also called "sensitive dependence on initial conditions." It only occurs in strongly nonlinear systems. To understand this concept, one should envision arbitrarily close orbits shooting off at divergent trajectories. This is not stochastic behavior. Chaotic behavior occurs in systems that are completely deterministic. The solution set of a chaotic system is called a *strange attractor*. Strange attractors are fractals.

That arbitrarily close deterministic solutions can diverge exponentially means that even simple systems, such as a compound pendulum, can behave unpredictably. The behavior of such systems becomes mathematically unpredictable because any error or perturbation, no matter how small, propagates until it overwhelms the underlying pattern. This is popularly known as the "butterfly effect," whereby a tiny force (such as the beating of a butterfly's wings) can have a dramatically disproportionate (nonlinear) effect (causing a proverbial tornado in Texas). The practical significance of chaotic behavior is that it defies prediction. Naturally, this goes right to the heart of any philosophy of science that takes as its goal the discovery of predictive laws.

"Self-organized criticality" is another concept that helps link dynamical systems and fractals [8,9]. Certain complex nonlinear dynamical systems exhibit self-organized criticality. "Criticality" refers to a marginally stable state toward which these systems spontaneously evolve. The classic model of this phenomenon is a sand pile to which sand is added one grain at a time. Eventually, the slope of the pile will reach a critical state—the angle of repose—after which the addition of more sand causes avalanches. Study of the avalanches indicates that they possess no natural scale, and they exhibit fractal statistics in both space and time. The avalanches allow the system to evolve back to a critical state, where the further addition of sand will cause more avalanches. Thus, after perturbation, the system evolves back to marginal stability. The fractal characteristics of the avalanches appear to explain several natural phenomena, including the fractal size–frequency distribution of geologic strata, the general fractality of erosional landscapes and hydrological systems [8, pp. 80–84], and the statistics of forest fires [76]. Some physical systems appear to unite all three concepts of fractals, chaos, and self-organized criticality: simulations of meandering rivers indicate that the system evolves to a critical state that oscillates between stability and chaos [86]. For the reader thinking of archaeological and human systems, self-organized criticality will inspire not only explanations for site stratigraphy and taphonomy, or settlement patterns, but also systemic models of historic patterns, such as the rise and collapse of early states or the pulsations of galactic polities [89].

3. Ancient Maya settlement

Ancient Maya settlement has been a subject of intense study for many years because of its implications for social complexity, urbanism, and cultural evolution. Modern analyses began in the 1950s with the mapping of Mayapán [51] and Willey's work in the Belize River valley [98]. Interest in settlement patterns has led to the creation of detailed maps of several Maya sites (Tikal, Guatemala: [24]; Copán, Honduras: [35]; Cobá, Quintana Roo, México: [36]; Calakmul, Campeche, México: [38]; Sayil, Yucatán, México: [78]; Dzibilchaltún, Yucatán, México: [87]; Seibal, Guatemala: [91]), as well as a number of atlases of regional settlement (Yucatán, México: [39]; Quintana Roo, México: [69]; Campeche, México: [70]; Chiapas, México: [75]). The overall corpus of data, however, remains spotty and incomplete, which makes it impossible to evaluate directly some hypotheses about settlement. The statistical tests used below, therefore, reflect the availability of appropriate types of data.

The body of literature in modern geography on the fractal characteristics of human settlement is significant and growing [10–16,18, pp. 144–149; 26,33, pp. 20–38; 34,59,60,97]. Several different kinds of modern settlements have been shown to be fractal in form. A number of investigators have studied the boundaries of modern cities and concluded that they are fractal curves that can be modeled by a process called diffusion limited aggregation (e.g., [10,14,15]). Others have discovered fractal patterns in the complex, maze-like streets of Tokyo [77]. Central Place lattices are ideal fractals [4,5,14, pp. 48–56]. Settlement hierarchies that obey the rank–size rule are fractal because the rank–size rule is a fractal relation [25,57]. The size–frequency relation for sites in many settlement patterns, including some Lowland Maya data, is a fractal (power-law) relation [19,27, pp. 17–19]. The segmentary internal structure of some traditional settlements is also fractal [18, pp. 144–149; 33, pp. 20–38; 34].

But not all settlement patterns are fractal. For example, the orthogonal grid pattern of an archetypal Roman city tends to be Euclidean rather than fractal, although its fractality depends on the details of the grid squares. Imagine a Roman grid centuriated with 10 insulae on a side. In our grid, which is clearly self-similar, from Eq. (2) we have a=100 squares, each scaled as s=1/10th the length of the side of the large square. Therefore, D=2, which is the correct Euclidean (and fractal) dimension. Thus, although the grid is self-similar, it is not fractal because the dimension is an integer not a fraction. If, however, one were to leave out certain grid squares from the pattern, it could quickly become a fractal like a "Sierpinski Carpet." Thus, for example, the internal grid layout of Teotihuacan might not be fractal (but see Oleschko et al., 72), while its irregular outline might well be fractal. In sum, the fractality of settlement patterns cannot be assumed. It must be demonstrated by argument and measurement. We will now see whether Maya settlement is fractal.

3.1. Inter-site settlement

On a regional level, the size-frequency distribution of Maya settlement is fractal. The calculation of the fractal dimension for a frequency distribution like this is simple. The fractal relation for such distributions is:

$$N(>r)=Mr^{-D} \quad (3)$$

where $N(>r)$ is the number of objects with a characteristic size greater than r, D is the fractal dimension, and M is a constant of proportionality [93, p. 42], cf. [92, p. 1921] [27, pp. 17–19]. The exponent D characterizes a specific distribution. It is a measure of the relative abundance of objects of different sizes. Thus:

$$D\propto -\frac{\ln(N(>r))}{\ln(r)} \quad (4)$$

To estimate D empirically, one plots the logarithm of the cumulative frequency [ln ($N(>r)$)] against the logarithm of size, ln(r). If this relation is linear, then the phenomenon is fractal. An estimate of the fractal dimension is provided by the "best fit" least squares linear regression line. The slope of the line is the negative of D.

As an example, let us examine some of Adams and Jones's data on site size based on courtyard counts [2, Table 1]. The true sizes of Maya sites are usually unknown because of the difficulty of performing survey in the rainforest.[1] For this reason, Adams and Jones [1,2] developed courtyard counts as a proxy for ranking sites by size. Although these data are far from ideal, they are the only rank–size data that exist for the Maya lowlands. The data should be taken with several caveats. For example, the various regions have not been identically surveyed, and it is clear that in all regions the smallest sites are under-represented [2, p. 307]. Nevertheless, the Río Bec region exhibits a fractal distribution. The relation is fractal because it is clearly linear (for the regression line, r^2=0.96, P=8.1 × 10^{-7}) (Fig. 2). The

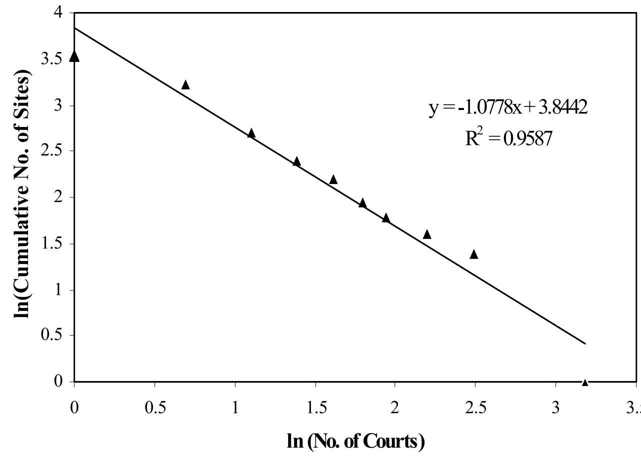

Fig. 2. Fractal distribution of settlement size based on courtyard counts, Rio Bec region (data from [1]).

fractal dimension D=1.08, which is comfortably close to the common and canonical value of 1. On the other hand, it should be noted that the Río Bec region is even today poorly surveyed. The bush is as thick and impenetrable as in any part of the Maya lowlands, and it is possible if not likely that sites of almost any magnitude might still be unknown. In addition, the Río Bec area has some sites, like Río Bec itself, that are unusually dispersed and that blend imperceptibly into dense rural settlement [71,90]. Therefore, the true sizes of sites are uncertain and ambiguous, and the reliability of courtyard counts as a proxy is even less certain.

Settlement hierarchies that obey the rank–size rule are fractal because the rank–size rule is a fractal relation [25,57]. The rank–size rule is an empirical observation that expresses the relationship between settlement size (population) and settlement rank (its numerical position in the series created by ordering all the settlements in the system from large to small). The idea that settlement size and rank have a systematic relationship was popularized by Zipf [100], who expressed it as:

$$P_{\mathrm{r}}=\frac{P_1}{r^k},r=1,2... \quad (5)$$

where P_{r} is the size of the settlement of r-th rank in the system and k is a constant, which is typically of the magnitude of 1, in the ideal case described by Zipf. The exponent k is calculated empirically by plotting the logarithm of rank against the logarithm of size: k is the slope of the best-fit line. The rank–size rule has been applied to archaeological data in various contexts (e.g., [25,48,57,73]). It is noteworthy that this is an empirical rule and does not depend on any sociological theory, like Zipf's "Law of the Least Effort." Rank–size has been used to analyze ancient settlement data from the Maya area and other parts of Mesoamerica

[1] The work of de Montmollin (e.g., [31,32]) is an exception to this rule: the topography and vegetation of highland Chiapas has allowed him to develop detailed settlement data for an extensive area. These Chiapas data remain anomalous, however, because of the small size of the settlements and presumed polities, and their multiethnic character, and their apparently peripheral relation to the Classic Maya lowlands.

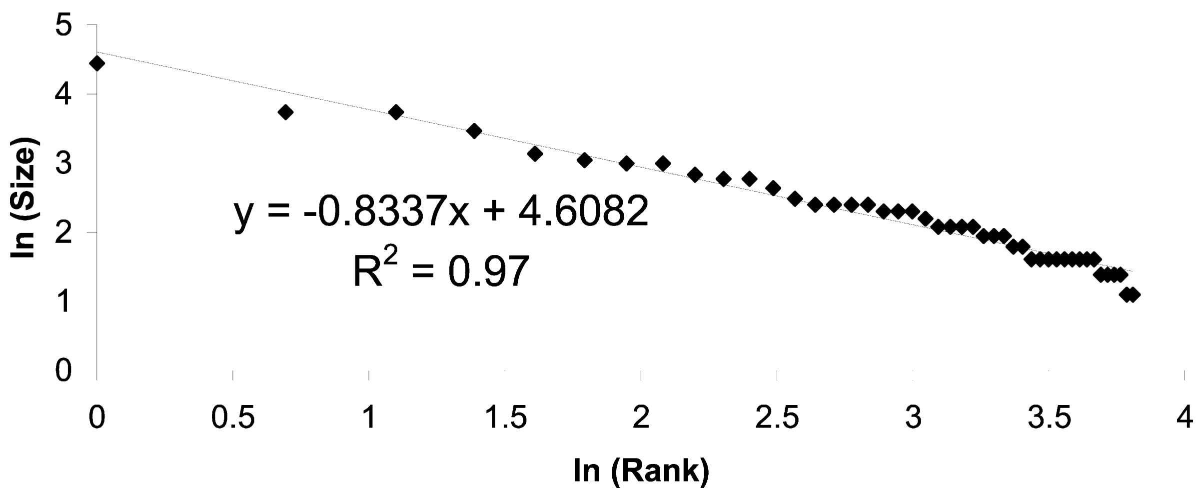

Fig. 3. The fractal rank–size relation for Adams [1] Data for the central Peten region.

[1,43,50,53,54]. There has also been study of the deviation of settlement systems from the expectations of the rank–size rule [48,100, pp. 374–375, 416–444], such as, for example, primate settlement systems that may be related to colonialism. Consequently, use of the rule is not merely a mechanistic exercise, but an informative model.

The fractal reformulation of the rank–size rule provides an important theoretical advantage over the original. The inherent self-similarity of the fractal relation means that a regional sample can be extrapolated to a whole settlement system [25,57]. The fractal dimension is related to the rank–size rule by:

$$D=-\frac{1}{k} \qquad (6)$$

[25, p. 62]. Thus, for k=1, Zipf's "classic" case, D=1 also. We used Adams's [1] data on site size for the Central Peten region to examine the fractal dimension of the rank–size relation for Maya settlement. We dropped the seven smallest sites to eliminate a "tail" in the plot that commonly occurs [57]. As one can see, the remaining sites form a highly linear, and thus a fractal, relation (Fig. 3). The exponent $k=-0.83$ and therefore D=1.2. This result is different from those reported for other regions [57].

To evaluate fully the fractality of regional Maya settlement, one should map every structure in some large region and then test to see if the pattern is self-similar and scale invariant across all scales of observation. Because this is a practical impossibility, we must resort to the examination of a variety of data sets that contain data at more restricted ranges of observation. Thus, we will instead look at the spatial distribution of sites in one region.

A geographical distribution of points or objects can be fractal when the pattern is self-similar. In the case of a pattern that is embedded in two (Euclidean) dimensions, self-similarity is commonly expressed as hierarchical, nested clusters, that is, clusters of clusters of clusters, ad infinitum. As with the other phenomena discussed, the fractality can be ideal or statistical. With empirical data sets, the fractality is usually statistical. This is the kind of fractal pattern that a distribution of sites across a landscape may exhibit.

The method used to evaluate the fractality of this kind of phenomenon, and to calculate the fractal dimension, is called the box-counting method. It is logically and methodologically linked to the Hausdorff–Besicovitch and capacity dimensions and normally provides an accurate estimate of them. The idea is this: one overlays a grid of squares on the points or curve to be measured, and one counts the number of boxes crossed by the curve or containing one or more points. The number of squares, N, required to cover the curve or points will depend on the size of the squares, s, so N is a function of s, or we can write $N(s)$. Now one reduces the size of the grid repeatedly, recording the two variables, N, and s. One plots the log of $N(s)$ vs. the log of s. If the slope of the least-squares regression line is d, then the fractal dimension is $D=-d$.

This procedure is difficult to perform by hand, but it is easily automated. A number of programs exist that perform some or all of the process. We used a program written by DiFalco and Sarraille called FD3 Version 0.3 [79] that was "inspired by" an algorithm devised by Liebovitch and Toth [58]. This program takes as input a series of coordinates for points, with one set of coordinates for one point on each line. The number of columns depends on the embedding dimension of the figure; our embedding dimension was 2, just x, y coordinates describing the locations of archaeological sites. Then it calculates the difference between the maximum and minimum values in the data set and uses this figure to determine the cell sizes. It begins with a single box, the sides of which are the length of the difference between the max and min values. This box is then divided into four cells by bisecting each side of the original box. The next division is into sixteen cells by subdividing each side into four segments. The next division is into sixty-four cells by a linear division of each side into eight segments, and so on. To accelerate computation, the program shifts and rescales the data set. Logarithms of base 2 are used in the calculation of dimension. The base of the logarithms does not affect the dimension because it is the ratio of two logarithms, which is the same regardless of the base.

To estimate the fractal dimension of Maya settlement on a regional scale, we chose the part of the Maya area that has been most completely surveyed: the state of Yucatán, México. We combined the data in the *Atlas Arqueológico del Estado de Yucatán* [39] with the newer supplementary data in Dunning [32, Table 5-1] to create a single list of 1107 sites within UTM Zone 16 in the state of Yucatán.

The contemporaneity of all these sites is an issue of concern. Unlikely though it may seem, we believe that all of these sites almost certainly had some level of occupation during the Late and Terminal Classic period, as represented mainly by the Cehpech (and to a lesser extent, the Sotuta) ceramic complexes [82]. Late and Terminal Classic settlement is ubiquitous in the region. This even applies to sites that are *mainly* occupied during other phases. Thus, Mayapán, the largest Late Postclassic site, has a Late Classic component represented by occasional potsherds and some Puuc style building stones that were reincorporated in the later architecture. Similarly, Komchen, a major Formative site, also has Late Classic remains [3]. Some sites are little investigated or poorly known. We judge, however, that it is extremely unlikely that any significant number of sites lack Late/Terminal Classic occupations. If a small number of sites are not contemporaneous, that fact is unlikely to alter significantly the statistical outcome of the estimate we are making.

Possibly more important is the lack of information from those parts of southern and eastern Yucatán that have not been surveyed extensively. There is little to be done about those lacunae now except to note that some investigators do believe that some of the lacunae are real gaps in settlement, rather than merely gaps in survey coverage (Bruce Dahlin, personal communication).

We converted the UTM coordinates from both works to full numeric coordinates and used them as Cartesian *x, y* coordinates for input to the FD3 program. FD3 calculated a fractal (capacity) dimension of D=1.51.[2] Since this value is clearly a fraction, not an integer, the pattern is fractal.

Joyce Marcus and others have argued that Classic Maya settlement conforms to the pattern of hexagonal lattices predicted by Central Place theory for a hierarchy that maximizes administration [37,64–66]. Arlinghaus [4,5] has shown that Central Place lattices are ideal fractals. Therefore, a fractal settlement pattern is clearly not inconsistent with one formed by the socioeconomic forces invoked by Central Place theory. Marcus's application of Central Place theory does have detractors, most prominently, Peter Mathews ([67,68,80]; see also [44, pp. 16–17; 45, pp. 275–276]). Most of the dispute, though, has focused on the degree to which emblem glyph references reflect hierarchical relations among centers. Little discussion has addressed Marcus's use of Central Place theory itself. Nevertheless, Central Place theory remains attractive and relevant to Mayanists [50], and we believe it merits additional research. Our purpose here, though, is neither to debate the theory nor to assert that fractal analysis "proves" it, but only to emphasize the compatibility of these two settlement geometries, Central Place and fractal.

3.2. Intra-site settlement

The intra-site spatial organization of Maya architecture is also fractal. Maya residential settlement is a complex pattern that has proven difficult to describe or quantify. Archaeologists have found that Maya residential settlement consists of small groups of buildings around formal or informal plazas. The house groups or *plazuelas* often coalesce into clusters of different sizes; the clusters sometimes form larger clusters that are like neighborhoods or quarters. In existing descriptions of Maya sites, we are often provided counts and measurements of different kinds of buildings and sometimes the statistics of building densities or inter-building distances. However accurate, these descriptions leave much to be desired. Some indefinable yet fundamental quality of the pattern typically eludes us. We believe that the ineffable quality that leads to endless patterned variations on a theme is fractal self-similarity. This exhibits itself in the clusters of clusters of clusters of domestic buildings (e.g., [6, pp. 46–58; 19, pp. 127–148; 21,30, pp. 53, 244; 31, p. 8; 55, pp. 73–81; 99 pp. 81–83]). To demonstrate that this pattern is fractal, we will measure the fractal dimension of the buildings on a part of the map of the Late Postclassic site of Mayapán, Yucatán, México [51].

Although the density of residential settlement at Mayapán is higher than at typical Classic period sites, the overall patterns are otherwise similar. Despite extensive investigation no one has ever discovered a structure

[2] Cell sizes (in m), before shifting and rescaling, used in the calculation of dimension, followed by the number of occupied cells: 425300/1, 212650/2, 106325/6, 53163/19, 26581/63, 13291/178, and 6645/408. The R^2 of the regression is 0.996, indicating a highly linear relation.

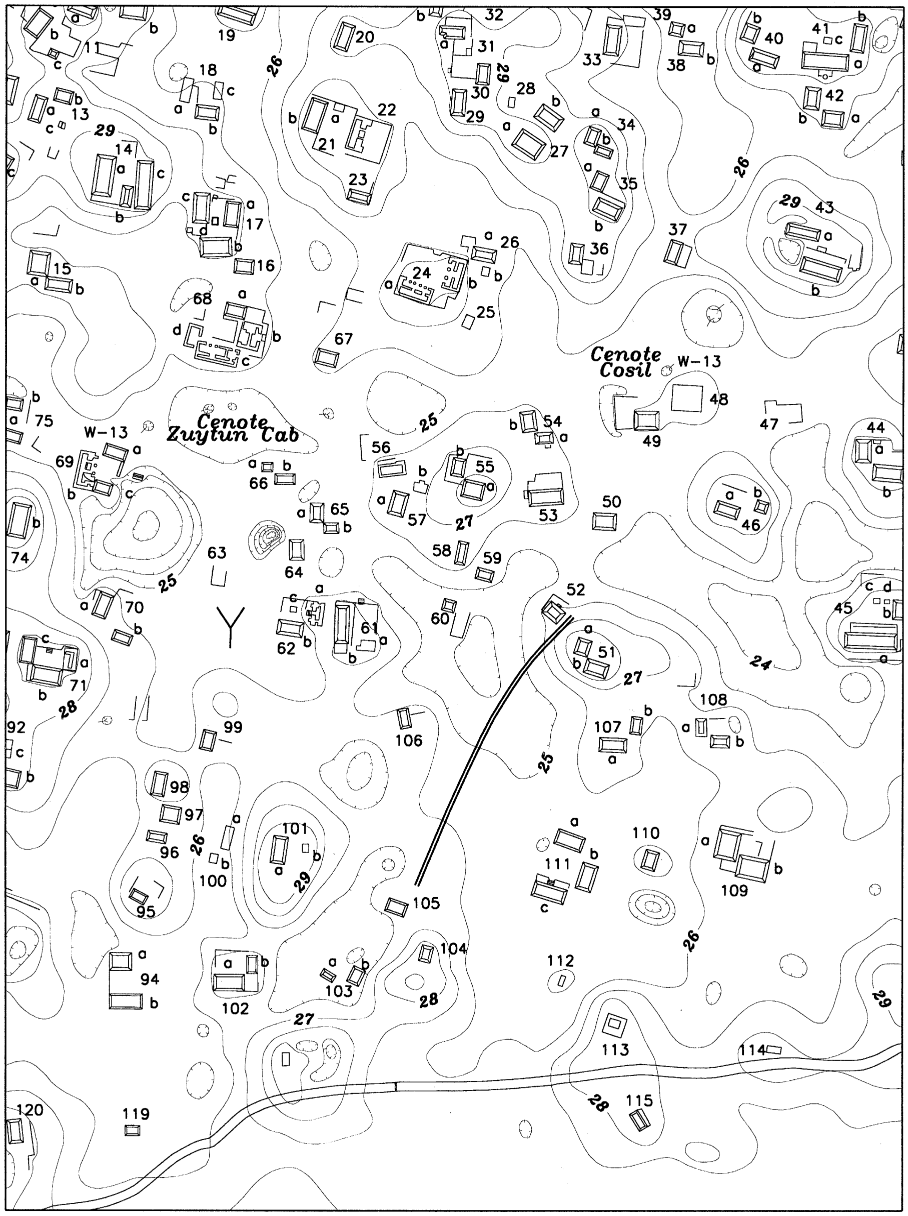

Fig. 4. Detail of square Y of the Carnegie Institution map of Mayapán. Drawn by Lynn A. Berg.

at Mayapán that is not Late Postclassic in date, although there is some evidence of earlier occupations in the form of both ceramics and re-used building stones. Thus, all the buildings on the map appear to be generally contemporaneous.

The pattern formed by the buildings on the map of Mayapán is embedded in two (topological) dimensions, and it must therefore be measured by the box-counting method. We created the input points for the FD3 program from a digitized copy of a portion of the Mayapán map [51] by "pointilizing" the structures in the selected area. We first traced a part of Square Y (Fig. 4) using a digitizing pad and a computer-aided drafting (CAD) program. Then we deleted all the elements (text, contour lines, grid lines, roads, etc.) that were not, apparently, buildings. Limiting ourselves to the constructions, then, we replaced each line with a series of points spaced 1 m apart. We exported this drawing information to a drawing exchange file (dxf) format and edited it with macros to extract x, y coordinates. This coordinate file was used as input to the FD3 program. The fractal dimension measured for this section of the Mayapán map is D=1.37, very clearly a fraction rather than an integer.

We also measured the fractal dimension of two other squares, K and R, using a commercial computer program entitled "Benoit", published by TruSoft International, Inc (reviewed in *Science* by Steffens [85]).

This program implements the standard box counting algorithm. It operates on an input file in the form of a black and white bitmap file for which it calculates the fractal dimension of the white pixels. We scanned these squares from the Mayapán map using a Hewlett–Packard Scanjet 5300c set at 600 dpi. We manipulated the bitmap image using Adobe Photo Deluxe Business Edition. First, we created a black and white image from the color scan. Then, we created a "negative" image from the bitmap file so that the architecture would appear as white pixels. We erased the labels, grid lines, contour lines, and so forth, and also cleaned up stray white pixels from dust on the scanner or foxing on the map.

The results are very similar to those from Square Y. Square R has a fractal dimension D=1.38 and Square K has a fractal dimension D=1.31. We also attempted to analyze a single bitmap image of Squares K and R. The program evidently could not process an image as large as 33 megabytes (the result of scanning at 600 dpi), possibly because of the architecture or configuration of our computer or operating system. Therefore, we reduced the resolution of the image to 300 dpi, which in no way affected the clarity of the delineation of the structures in the image. The resulting analysis estimated the fractal dimension of the two squares as D=1.33, which seems plausible because it falls between the estimates for the two squares taken individually. In total, we have measured the fractal dimension of 0.7279 km^2 of Mayapán (Square Y has an area of 0.2279 km^2). The whole site within the great wall measures approximately 4.2 km^2. We think it is beyond question that residential settlement at Mayapán is fractal with dimension $D \approx 1.35 \pm 0.05$.

Earlier in this paper, we discussed the Euclidean internal arrangement of some cities built on an orthogonal grid, and we noted that their exterior outline might nevertheless be a fractal curve. In contrast, at Mayapán the internal organization is fractal, while the perimeter, marked by the course of the 9 km long great wall that approximates the boundary of the community, is essentially a Euclidean curve; D for the wall is very close to 1.0.

We thought that the fractal dimension of residential settlement would vary from site to site because of local and regional variation in architecture, settlement, and geomorphology. As a test, we measured the fractal dimension of a small portion of Dzibilchaltún, a very large site near the north coast of Yucatán, México. Dzibilchaltún was occupied from the Middle Formative period through the Late Postclassic and into the Colonial period. More than 90% of the visible architecture, however, dates from the Late and Terminal Classic periods (Early Period II–Pure Florescent Period), during the Copo 1 and 2 phases [55, p. 39].

We evaluated the fractal dimension of a small area in the southwest part of Sheet L:L [87]. The area examined begins at the southwest corner of the map sheet and runs 550 m (in map units) north and 400 m east. This particular spot was selected rather arbitrarily because it was intermediate between the ceremonial center (we did not want to include civic–ceremonial architecture in the sample) and the sparsely populated hinterland. It also appears to a fairly representative slice of habitation. We scanned this section of the map sheet and treated the resulting bitmap file as described above. We had to decrease the resolution of the image to 300 dpi to reduce the file size. We were surprised to find that the fractal dimension of this area was D=1.23, closer to the value for Mayapán than we expected. The lower value probably reflects the somewhat different and more dispersed character of the clusters at Dzibilchaltún. The distinctive settlement pattern at Dzibilchaltún is probably caused by a combination of sociological and geomorphological factors. For example, the settlement at Mayapán is quite dense, perhaps because everyone felt a need to live within the great defensive wall. The low but steep relief at the site also significantly influences settlement: almost all the buildings are on high ground (residual karst ridges and knolls) [19]. At Dzibilchaltún there is no wall, and settlement feathers lightly and indefinitely to the horizon. There is also no topographic relief. The terrain is flat and settlement patterns are not constrained in the same ways they are at Mayapán. The value of the fractal dimension reflects the overall effect of these differences.

The discovery of the fractality of Maya residential settlement is an important advance. First, the fractal dimension provides a quantitative characterization of a pattern that has been difficult to describe efficiently. As we have seen, the value of D varies among different kinds of sites. Further research will show in more detail how this parameter varies from one type of site to another. Second, we can now assert that Maya intra-site settlement patterns are self-similar. This is a key concept that coincides perfectly with an analysis of Maya settlement patterns as consisting of nested groups of groups.

This settlement pattern reproduces quite accurately a social system in which the family is a microcosm of the lineage, which in turn is a reduced version of the clan, which itself replicates the state. The self-similarity of the family-lineage–clan–state hierarchy is characteristic of segmentary lineage systems, segmentary states [83], and galactic polities [88,89], which are among the prevailing models of Maya socio-political organization [28, p. 146; 29, p. 823]. For example, the idea of the galactic polity is based on the state as a mandala, and a mandala is structurally self-similar.

> At a surface level the cosmological account gives a magnificent picture of the exemplary center pulling

> together and holding in balance the surrounding polity. But we can properly appreciate in what manner the center attempted to hold the remainder—the centripetal role of the center—only after we have properly understood the decentralized locational propensity of the traditional polity and its replication of like entities on a decreasing scale; in other words, only after we have grasped the structure of the galactic constellation, which is a far cry from a bureaucratic hierarchy in the Weberian sense [89 p. 266].

The structure of segmentary lineage systems is also self-similar, as Southall's description of the structure of the Alur segmentary state reminds us.

> (5) Several levels of subordinate foci may be distinguishable, organized pyramidally in relation to the central authority. The central and peripheral authorities reflect the same model, the latter being reduced images of the former. Similar powers are repeated at each level with a decreasing range, every authority has certain recognized powers over the subordinate authorities articulated to it, and formally similar offenses differ in significance according to the order of authorities involved in them [83, p. 249].

This is a description of a self-similar structure. The overall structure is composed of structural replicas at increasing smaller scales.

This type of self-similarity is also evident ethnologically among the Maya. Vogt calls it "replication." "The patterned aspect of Zinacanteco culture that impresses me most is the systematic manner in which structural forms and ritual behaviors are replicated at various levels in the society …" [94, p. 571].

> Structural replication is manifested in both the social and ritual systems. In the social system, the settlement pattern takes the form of an aggregate of aggregates ranging from the house, or houses, of a single domestic group up to the total municipio with its ceremonial center forming the focal point of tribal activities. Similarly, the social structure consists of units of increasing scale: the domestic group occupying a house compound; the sna composed of one or more localized patrilineages; the waterhole groups composed of two or more snas; the hamlet; and finally the ceremonial center. All social levels are found in the large parajes, but smaller parajes may, for example, consist of a single waterhole group [94, p. 572].

In short, the fractal settlement pattern is a physical outline of the self-similar structure of Maya society as deduced from historical and ethnological sources.

It is important to note, however, that nothing inherent in the fractality of the ancient settlement patterns tells us that the ancient descent system was necessarily lineal. Gillespie [40,41] has recently argued that the Maya had bilateral or cognatic descent groups called "houses." Presumably, any kind of descent group can produce a fractal settlement pattern provided the rules of descent and the demography generated the proper scaling relations among the sizes of the nested units of organization. To distinguish among the various kinds of settlement patterns produced by different types of descent groups, one should conduct ethnological or ethnohistorical studies of their genealogy, demography, and settlement. As an alternative, social group patterns like these could be usefully studied through simulations using fairly simple rules. It is possible that different rules of descent would tend to produce different scaling relations among the hierarchically ordered social units. Nevertheless, we believe all the ethnohistorical and ethnological evidence points to lineal social organizations for the various Maya subcultures; we find the evidence for Maya "houses" presented by Gillespie to be weak and unpersuasive. We believe the prevailing unilineal and bilineal models of Maya descent have much stronger evidentiary foundations [19, pp. 484–571]. None of the arguments that follow, however, depend upon the Maya having possessed any particular type of descent group.

4. Inferences

What can we learn about the ancient Maya from the fractal nature of their settlement patterns? The fact that Maya settlement is fractal carries important implications for our understanding of the society that produced them. First, one can infer that traditional Maya social structure is fractal just like the ancient settlement patterns. Second, one can learn about the dynamics of such fractal societies by modeling them using fractal processes. Zubrow [101] has simulated ancient Maya settlement patterns using fractal methods. It also appears that Maya settlement can be modeled using a process called random fractal curdling with a probability of 0.5 [18, pp. 92–101; 63]. Such modeling can lead to greater insight into the underlying dynamics of the social processes that created the settlement patterns. It also seems reasonable to infer that some aspects of the ancient Maya social system were chaotic or self-organized critical. Chaos theory and self-organized criticality also can offer new insights into ancient social processes.

For example, Roberts and Turcotte [76] have reviewed the well known "forest fire" model of self-organized criticality. The model works like this. Let there be a grid of points. At each time step or iteration of the model, either a tree or a match (i.e., a spark) is dropped on a randomly selected point. If a tree is dropped on a vacant point, it takes root. If a spark drops on a vacant point, nothing happens. If a match is dropped on a tree, it catches fire, and then it ignites any trees on adjacent points. Those trees in turn ignite any trees adjacent to them, propagating the fire throughout the extent of the contiguous cluster of trees. As this process of growth and destruction is repeated through many time steps, it creates clusters of trees with a fractal size–frequency distribution. As the clusters ignite and burn, a fractal size–frequency distribution of forest fires is generated. The sizes of the forest fires are analogous to the sizes of the avalanches in the sand pile model described earlier. When the frequency of sparks is high, the process is dominated by many small fires: clusters tend to burn before they grow to large size. When the sparking frequency is low, the size–frequency distribution is dominated by a few large fires that sweep across the entire grid: large clusters of trees develop before finally burning. Roberts and Turcotte show that the fractal dimensions of the size–frequency distribution of forest fires generated by the model closely match the fractal dimensions of actual, empirical forest fire statistics. Although Roberts and Turcotte do not mention it, empirical vegetation patches are spatial fractals [47, pp. 119–133]. The fractality and dimensions of the vegetation patches may well be related to forest fire dynamics as well as other factors.

Roberts and Turcotte [76] go on to analyze the intensity–frequency distributions of wars using various historical data sets representing two different measures of the intensity of war. They find that the fractal dimensions of the frequency relation for the intensity of war are similar to those for forest fires. They conclude that war may also be a result of a self-organized critical process in a metastable world system. They suggest that war can spread like a conflagration through groups of countries. The values of fractal dimension for the key data sets range from D=1.27−1.54, the same as those for some Maya settlement described here.

Notwithstanding earlier theories that the Maya were peaceful farmers and astronomers, archaeologists now believe that the Maya lowlands were plagued by chronic internecine war. The original, and still compelling, evidence for brutal warfare among the Maya was the murals of Bonampak, Chiapas, discovered in the late 1940s. It is now known that a number of sites, both Classic and Postclassic, were fortified. Even more significant evidence for warfare comes from the monumental inscriptions. They speak incessantly of conquest, capture, and domination. David Webster is the most persistent and prominent student of Maya warfare. In detailed recent reviews (e.g., [95,96]), he argues persuasively for the significance of warfare in Maya sociopolitical evolution. It is clear that Maya warfare occurred on many scales, from minor raids aimed at capturing prisoners to major conflicts between large states. There can be no reasonable question that warfare influenced ancient Maya demographics and settlement patterns across a range of scales.

We can document, then, fractal patterns of Maya settlement, some of which have the fractal dimensions of the same magnitude as those for forest fires and wars, and archaeological, iconographic, and epigraphic evidence for chronic, incessant warfare at many scales throughout Maya history. The settlement patterns and the historical patterns fulfill all of the criteria and expectations of a self-organized critical model.

To these facts, we must add Joyce Marcus's [66] theory of "dynamic cycling." She argues that Maya polities passed through repeated cycles of geographical growth and fragmentation. These observations have attracted widespread interest (e.g., [30, p. 53; 31, pp. 249–267; 49,84, p. 59]). Marcus describes historically [66, pp. 157–170] the periodic coalescence of regional states followed by their collapse into smaller constituent polities. In the terminology of current systems theory, her model is a qualitative description of self-organized criticality. We do not believe that this is an accident. We believe that analysis of the archaeological record will eventually show that early states were self-organized critical systems that were far from equilibrium. A nonlinear system can react dramatically to a disproportionately small impetus. This would explain why some early states were evidently susceptible to major changes in the face of minor forces. Therefore, it may not be necessary for archaeologists to identify a major change in politics, technology, demography, or environment to explain the sudden growth or collapse of a state: a minor perturbation may be all that is necessary because it acts like a grain of sand that causes an avalanche.

We do not claim to have proven that Maya society was a self-organized critical system, but rather we suggest this as an hypothesis to be investigated through further archaeological research. The verification or falsification of the model will come from the statistics of Maya warfare, and through the archaeological and epigraphic documentation of its extent and intensity throughout Maya history and prehistory. Similarly, if the sizes of Maya polities exhibit fractal statistics through time and space, it would be virtual proof of Maya political evolution as a self-organized critical system. This then is the type of explanation that we can expect from nonlinear analyses of archaeological remains.

5. Conclusion

Fractal theory provides a simple and coherent way of understanding very complex phenomena. Fractal phenomena appear to be widespread in archaeology. As we have seen, fractal analysis is not an isolated statistical method. Fractal analysis includes a large range of analytical methods for study of the irregularity and complexity. Many of those methods are related to better-known statistics (such as the Zipf distribution, the rank–size rule, or, in geomorphology, the variogram). These methods are the ones, and often the only ones, that are mathematically appropriate for fractal phenomena. Traditional statistical methods usually will not yield consistent estimates of fractal parameters. Therefore, it is indispensable for archaeologists to recognize fractals and to analyze them correctly. In this article, we have not only shown that Maya settlement is fractal, but we have also illustrated the types of systems theoretical explanations that may be responsible for the patterns.

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