Practice Problems for Exam 4

1. (a) $\sin 2$. L'Hopital's rule does not apply (the limit does not have an indeterminate form). The limit can be evaluated by substitution.

   (b) 0 (Use L'Hopital's rule and then do some algebra to simplify before substituting $x = 0$.)

2. (a) 10.5 miles; (b) to know we actually have an over- and under-estimates of the distance traveled; (c) 70 times in the 1 hour period

3. Using $\Delta x = 0.6$, we have $LHS(5) = \frac{0.6}{\ln 5.0} + \frac{0.6}{\ln 5.6} + \frac{0.6}{\ln 6.2} + \frac{0.6}{\ln 6.8} + \frac{0.6}{\ln 7.4}$; (b) Overestimate since $\frac{1}{\ln x}$ is a decreasing function

4. (a) GMIN $= 4e$, GMAX $= \frac{6}{\ln 1.5}$; (b) Find overestimate by computing LHS for $\displaystyle\int_3^{2e} f(x)\,dx$ and adding it to RHS for $\displaystyle\int_{2e}^{8} f(x)\,dx$; Find underestimate by computing RHS for $\displaystyle\int_3^{2e} f(x)\,dx$ and adding it to LHS for $\displaystyle\int_{2e}^{8} f(x)\,dx$. (c) 57.14 using $n = 300$ subdivisions for each of the four sums

5. F is increasing and concave down, with a max between 0 and x_1; then decreasing and concave down with an inflection point at x_1; then decreasing and concave up, with a minimum at x_2. From x_2 to x_3 and beyond, F is increasing and concave up.

6. (a) $\frac{1-e^{-2k}}{2k}$, (b) When $k = 1$, average value is about 0.432, which is the height of a box with base 2 whose area is the same as $\int_0^2 e^{-x}\,dx$, (c) less than; e^{-x} is concave up.

7. (a) 1, (b) 4, (c) positive since $\int_3^7 f'(t)\,dt$ is around -2.5, so $f(7) \approx 4 - 2.5 > 0$

8. (a) The average value of f on [0,30] is larger than the average of $f(0)$ and $f(30)$. The former is the height of a box with base $[0, 30]$ and area $\int_0^{30} f(x)\,dx$, while the latter is halfway between $f(0)$ and $f(30)$. (b) $LHS(5) < LHS(10) < \int_0^{30} f(x)\,dx < RHS(10) < RHS(5)$

9. $\frac{1}{2}x^2 + 2\sqrt{x} - \frac{1}{2}\sin 2x + c$

10. $\frac{-5}{e^4} + \frac{5}{e} + \frac{63}{64}$