

Practice Problems for Exam 4

1. For each limit below, use l'Hopital's rule to find the limit if possible. Math grammar counts! If l'Hopital's rule does not apply, explain why not, and evaluate the limit using another method (explain your method!).

(a) $\lim_{x \rightarrow 1} \frac{\sin(2x)}{x}$

(b) $\lim_{x \rightarrow 0} \frac{\sin x}{x^{1/3}}$

2. Roger decided to participate in a bicycle race up Pike's Peak. On the steepest section of the road, Roger's wife Sylvia clocked his speed every 10 minutes, and recorded the following data:

Time (minutes)	0	10	20	30	40	50	60
Speed (mph)	18	14	13	11	9	5	4

- (a) Assuming Roger's speed is strictly decreasing, what is your best estimate for the length of the steepest section of the road?
- (b) Explain why we need the assumption of strictly decreasing speed in the previous part.
- (c) How many times would Sylvia need to record Roger's speed during the 60 minute steep stretch if you wanted your best estimate of the length of the steep section to be within 0.1 mile of the actual distance?

3. Consider the definite integral $\int_5^8 \frac{1}{\ln x} dx$.

- (a) Write down the terms of the left-hand Riemann sum with $n = 5$ that could be used to approximate $\int_5^8 \frac{1}{\ln x} dx$. Do not evaluate the terms or the sum.
- (b) Would your Riemann sum from the previous part over-estimate or under-estimate the actual value of the integral? Justify your answer.

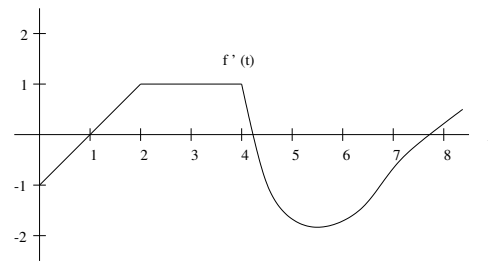
4. Let $f(x) = \frac{2x}{\ln(x/2)}$.

- (a) Find the global minimum and maximum of f on the interval $[3,8]$.
- (b) Carefully explain how you could find an over-estimate and an under-estimate of $\int_3^8 f(x) dx$ using left and right Riemann sums.
- (c) Use your method from the previous part to estimate $\int_3^8 f(x) dx$ with error at most 0.1.

5. The graph of $f(x)$ is shown on page 285, #12. Sketch a graph of $F(x)$ where $F'(x) = f(x)$ and $F(0) = 1$. Identify local extrema and inflection points of F .

6. (a) Find the average value of e^{-kx} on the interval $[0, 2]$. Your answer will involve k .
- (b) Using $k = 1$, draw a picture and write one sentence that together explain the graphical interpretation of your answer.
- (c) Is your answer to part (a) greater than or less than the average of 1 and e^{-2} ? Explain your answer graphically.

7. The graph of the derivative f' of a function f is shown below. Assume $f(0) = 3$.



- (a) Find $\int_0^3 f'(t) dt$.
- (b) What is $f(3)$?
- (c) Is $f(7)$ positive or negative? Justify your answer.
8. Daily sales of widgets are increasing, but at a decreasing rate. Assume $f(t)$ gives the total sales (in dollars per day) of widgets t days after the product is introduced.
- (a) Which is greater, the average value of f on $[0, 30]$ or the average of $f(0)$ and $f(30)$? Use a picture to explain briefly.
- (b) Assuming all the Riemann sums are estimates for the definite integral $\int_0^{30} f(t) dt$, put the following values in order from least to greatest: LHS(5), LHS(10), RHS(5), RHS(10), $\int_0^{30} f(t) dt$.
9. Find the antiderivative: $\int (x + \frac{1}{\sqrt{x}} - \cos 2x) dx$.
10. Evaluate the definite integral exactly (do not give a decimal approximation): $\int_1^4 \left(\frac{5}{e^r} + \frac{3}{r^4} \right) dr$.