Practice for Test 3

1. On February 10, 1990, high tide in Boston was at midnight. The water level at high tide was 9.9 feet; later, at low tide, it was 0.1 feet. Assuming the next high tide is at exactly 12 noon the water level, y, in Boston Harbor can be estimated by

$$y = 5.0 + 4.9\cos(\pi t/6) \text{ feet},$$

where t is hours since midnight on February 10th.

   (a) What does dy/dt represent, in terms of water level? Be sure to include units.

   (b) For $0 \le t \le 24$, when is dy/dt equal to 0? Explain what it means (in terms of water level) for dy/dt to be 0. Be as specific as possible. [Hint: Use a graph of y to help answer the questions.]

2. For what values of x is the graph of $y = xe^{-x}$ increasing? concave down? Fully justify your answers algebraically.

3. Use the values in the table to compute the requested derivatives.

x	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
0	3	1	-2	5
2	4	-1	5	3
4	0	-2	6	1/2

   (a) $f'(2)$ if $f(x) = \dfrac{g(x)}{h(x)}$

   (b) $f'(2)$ if $f(x) = \sqrt{g(x)}$

   (c) $f'(2)$ if $f(x) = h(2x^2 - 2x)$

   (d) $f'(2)$ if $f(x) = \tan(\pi x) \cdot g(x)$

4. Assume the function f is defined and continuous for all real x. Let $x_1 < x_2 < x_3$. Sketch a possible graph of $y = f(x)$, assuming:

   - $f' > 0$ on $(-\infty, x_1)$, $f'(x_1) = 0$, $f' > 0$ on (x_1, x_3), $f'(x_3) = 0$, and $f' < 0$ on (x_3, ∞), and
   - $f'' < 0$ on $(-\infty, x_1)$, $f''(x_1) = 0$, $f'' > 0$ on (x_1, x_2), $f''(x_2) = 0$, $f'' < 0$ on (x_2, ∞).

5. Find the equation of the line tangent to the graph of $x \ln y + 3y^2 = x$ at the point $(3, 1)$.

6. (a) Find the local linearization (aka, the tangent line) of $\sqrt{1+x}$ near $x = 0$.

   (b) Use the local linearization to approximate $\sqrt{1.1}$. Is the approximation larger or smaller than the actual value of $\sqrt{1.1}$. Explain your answer graphically.

7. For each limit below, use l'Hopital's rule to find the limit if possible. Math grammar counts! If l'Hopital's rule does not apply, explain why not, and evaluate the limit using another method (explain your method!).

   (a) $\displaystyle\lim_{x\to 1} \frac{\sin(2x)}{x}$

   (b) $\displaystyle\lim_{x\to 0} \frac{\sin x}{x^{1/3}}$

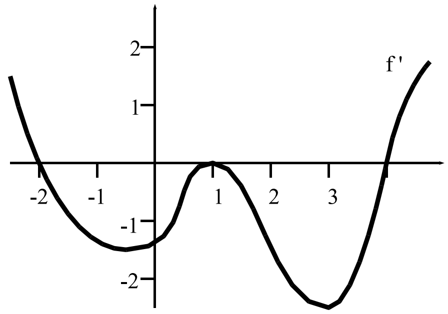
8. Using the chain rule, develop a formula for $\dfrac{d}{dx}(\arcsin x)$. (Don't just give the derivative of $\arcsin x$. Derive it!)

9. A 13-foot ladder resting on horizontal ground is leaning against a vertical wall when its base starts to slide away from the wall. At the time the base is 12 feet from the wall, the base is moving at a rate of 10 feet per second.

(a) How fast is the top of the ladder sliding down the wall then?

(b) How fast is the area of the triangle formed by the ladder, wall and ground changing?

10. The graph of $y = f'(x)$ is shown below.



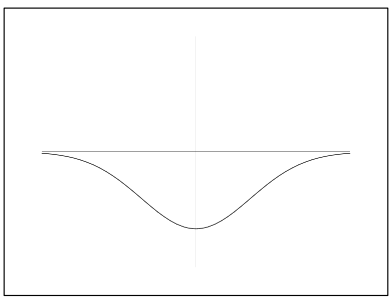
(a) At what values of x does $f(x)$ have critical points?

(b) For each critical point, does f have a minimum, a maximum, or neither at the point? Briefly justify your answers.

(c) At what values of x does f have an inflection point? Explain briefly.

11. Suppose a rectangular beam is cut from a cylindrical log of radius 30 cm. The strength of a beam of width w and height h is proportional to the product of the width and the square of the height. Find the width and height of the beam of maximum strength. Fully justify your answer.

12. A function $y = g(x)$ has a derivative that is given below for $-2 \leq x \leq 2$.



(a) Sketch a graph of g''.

(b) In one or two clear, concise and complete sentences, describe the behavior of g on the interval $-2 \leq x \leq 2$.

(c) Sketch a possible graph of g.

13. Huey makes skateboards in his garage and sells them for \$50 each. He estimates that his cost for building q boards in a month is given (in dollars) by

$$C(q) = .03q^3 - 2.25q^2 + 58.64q + 300.$$

Assuming Huey can sell all the skateboards he makes, what is his maximum monthly profit? Fully justify your answer. (Profit = Revenue - Cost = $50q - C(q)$.)

14. A square-bottomed box with no top has a fixed volume V. What dimensions minimize the surface area?