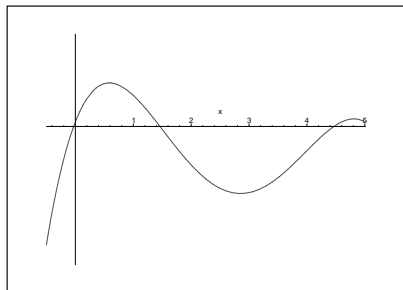
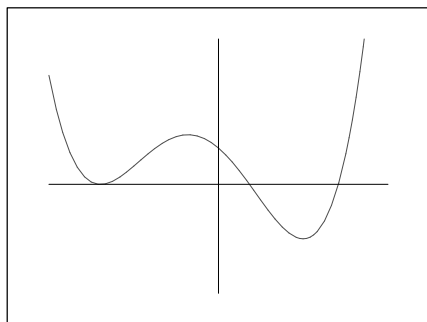


Review Problems for Test 2

- Estimate the derivative of $f(x) = x^{\sin x}$ at $x = 1$. Be sure to show your work and/or explain your reasoning. [An explanation such as ‘I used the *blah* function on my calculator’ will receive no credit.]
- The graph of a function f is shown below. Sketch a graph of $f'(x)$.



- Suppose $C(r)$ is the total cost of paying off a car loan borrowed at an annual interest rate of $r\%$.
 - What are the units of $C'(r)$?
 - What is the sign of $C'(r)$? Explain briefly.
 - What is the practical meaning of $C'(5.0)$? Be sure your statement is clear!
- The value of Andrew’s 1997 Sebring convertible can be approximated by the function $V(t) = 25(0.85)^t$ thousands of dollars, when t is the time, in years, since the date of purchase.
 - Evaluate and interpret $V(4)$.
 - Find an expression for $V'(t)$, including units.
 - Evaluate and interpret $V'(4)$.
- There is new data that suggests that the annual rate of growth of the world population peaked last year. Let $P(t)$ be the number of people alive in year t . Sketch a graph of P and its derivatives. Explain why your graphs look the way they do.
- The graph of f' (*not* f) is given. At which of the marked values of x is



- $f(x)$ greatest?
- $f(x)$ least?
- $f'(x)$ greatest?
- $f'(x)$ least?
- $f''(x)$ greatest?
- $f''(x)$ least?

- A cable is made of an insulating material in the shape of a long, thin cylinder of radius r_0 . It has electric charge distributed evenly throughout it. The electric field E at a distance r from the center of the cable is given by

$$E = \begin{cases} kr, & \text{for } r \leq r_0 \\ k\frac{r_0^2}{r}, & \text{for } r > r_0 \end{cases}$$

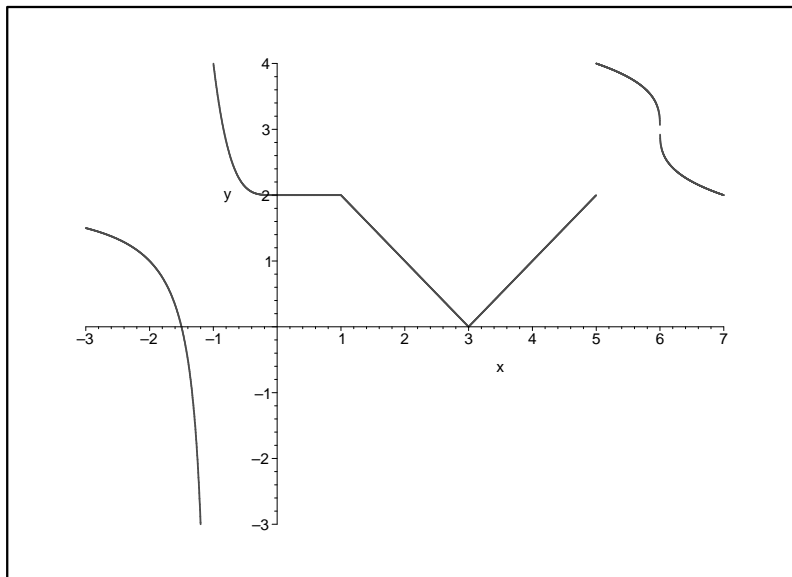
- (a) Sketch a graph of E as a function of r .
- (b) Is E continuous at r_0 ?
- (c) Is E differentiable at r_0 ?

| x | $g(x)$ | $g'(x)$ | $h(x)$ | $h'(x)$ |
|-----|--------|---------|--------|---------|
| 0 | 3 | 1 | -2 | 5 |
| 2 | 4 | -1 | 5 | 3 |
| 4 | 0 | -2 | 6 | 1/2 |

8. Use the values in the table to compute the requested derivatives.

- (a) $f'(2)$ if $f(x) = \frac{g(x)}{h(x)}$
- (b) $f'(2)$ if $f(x) = \sqrt{g(x)}$
- (c) $f'(2)$ if $f(x) = h(2x^2 - 2x)$

9. A graph of f is shown below.



- (a) At what values of x is f not continuous?
- (b) At what values of x is f not differentiable?
- (c) Estimate the following, based on the graph:
 - i. $\lim_{x \rightarrow 3} f(x)$
 - ii. $f(3)$
 - iii. $\lim_{h \rightarrow 0^+} \frac{f(3+h) - f(3)}{h}$
 - iv. $\lim_{h \rightarrow 0^-} \frac{f(3+h) - f(3)}{h}$
- (d) Explain how each of your answers in the previous part relates to the continuity and differentiability of f at $x = 3$.

10. Dr. Paul Reillo, one of our previous forum speakers, suggested that world rainforests were disappearing at a rate of about 1% per year. Figures vary by source, but rainforests were estimated to cover approximately 8.9 million square kilometers in 2000.

- (a) Find an exponential formula for a function f that would estimate the area of land covered by rainforest t years after 2000, assuming the estimates above.
- (b) Find $f'(t)$.
- (c) Find $f'(3)$ and interpret your answer in practical terms. Be sure to include units.
- (d) Find $f''(t)$.
- (e) Find $f''(3)$ and interpret your answer in practical terms. Be sure to include units.
- (f) Estimate $f^{-1}(8)$. What does your answer mean in practical terms?
- (g) What is the practical meaning of the derivative of f^{-1} at 8?
- (h) Find a careful estimate the derivative of f^{-1}
11. Is the function $f(x) = \begin{cases} x^2 \sin(1/x) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$ differentiable at $x = 0$? Justify your answer.
12. The height of a sand dune (in centimeters) is represented by $f(t) = 700 - 3t^2$, where t is measured in years since 1995.
- (a) Find $f(5)$ and interpret your answer, using units, in terms of the sand dune.
- (b) Find $f'(5)$ and interpret your answer, using units, in terms of the sand dune.