

Related Rates

1. A Japanese beetle infestation is spreading from the center of a small town. Since beetles fly in all directions, we assume the region they cover is circular. Suppose the radius of this circular region is increasing at a rate of 1.5 miles per year. Determine the rate of change of the area of infestation when the radius is 4 miles.
2. An airplane is flying at a height of 5 miles and is traveling at a speed of 400 mph toward Miami. When its horizontal distance from Miami is 13 miles, how fast is the line-of-sight distance between the plane and Miami changing?
3. Suppose that a man 6 feet tall is walking at a speed of 8 feet/sec away from a street light, which is atop an 18 foot pole. How fast is the tip of his shadow moving along the ground?
4. Suppose that a gas balloon is being filled at the rate of 100π cubic centimeters of helium per second. At what rate is the radius of the balloon increasing when the radius is 10 cm? At what rate is the radius of the balloon increasing when the radius is 100 cm? Explain why the relative sizes of your two answers are plausible.
5. A 13 foot ladder resting on horizontal ground is leaning against a vertical wall when its base starts to slide away from the wall. At the time the base is 12 feet from the wall, the base is moving at the rate of 10 feet per second. How fast is the top of the ladder sliding down the wall then? How fast is the area of the triangle formed by the ladder, wall and ground changing?
6. Assume that a balloon is being inflated at a constant rate of 8 cubic centimeters per second, and the balloon has no air in it initially. One second after the inflation of the balloon begins, one mosquito lands at the north pole of the balloon, and another lands on the equator. How fast are the mosquitoes parting company 4 seconds after landing? [The volume of a sphere of radius r is $4\pi r^3/3$.]
7. The ends of a water trough 8 feet long are equilateral triangles whose sides are 2 feet long. If water is being pumped into the trough at a rate of 3 cubic feet per minute, find the rate at which the water level is rising after 2 minutes.
8. Suppose that a drop of mist is a perfect sphere and that, through condensation, the drop picks up moisture at a rate proportional to its surface area. Show that under these circumstances the drop's radius increases at a constant rate.
9. A tanker accident has spilled oil in Pristine Bay. Oil-eating bacteria are gobbling bacteria at a rate of 5 ft³/hr. The oil slick has the form of a circular cylinder. When the radius of the cylinder is 500 ft, the thickness of the slick is 0.01 ft and decreasing at a rate of 0.001 ft/hr. At what rate is the area of the slick changing at this time? Is the area of the slick increasing or decreasing?
10. A stone is dropped into a lake, causing circular waves whose radii increase at a constant rate of 0.6 meters per second. At what rate is the circumference of a wave changing when its radius is 4 meters?
11. American Flight 1003 is traveling from Minneapolis to New Orleans, and United Flight 366 is traveling from Los Angeles to New York. Both flights are at 33,000 feet, and the flight paths intersect over Ottumwa, Iowa. At 1:30 p.m. Central Time the American flight is 32 nautical miles (horizontally) from Ottumwa and is approaching it on a heading of 171 degrees at a rate of 405 knots. The United flight is 44 nautical miles from Ottumwa and is approaching it on a heading of 81 degrees at a rate of 465 knots.
 - (a) At this instant, how fast is the distance between the planes changing?
 - (b) How close will the planes come to each other? Will they violate the FAA's minimum separation requirement of 5 nautical miles? Will they collide if Air Traffic Control does not act to separate them?
 - (c) What time will it be at the time of closest approach? Is there enough time for the ATC to take appropriate action?
12. During a storm, the combination of rainfall and runoff from the gutters causes water to pour into your backyard 'pond' (it's only a pond when it rains) at a rate of 2 cubic inches per second. If the depth of the pond is uniformly 1/2 inch, how quickly is the pond covering your lawn? (That is, how quickly is the surface area of the pond increasing?)

Related Rates Answers

1. 12π square miles per year
2. $-1300/3$ mph
3. 12 ft/sec
4. $1/4$ cm/sec; $1/400$ cm/sec
5. -24 ft/sec; -119 square feet/sec
6. straight line: 0.200 cm/sec; arclength: 0.222 cm/sec
7. $3/(8\sqrt[4]{3})$ ft/min
- 8.
9. 78,037 square feet per hour
10. 1.2π m/sec
11. (a) 614.3 knots, (b) 4.768 miles (c) 5.27 minutes later
12. 4 square inches per second