Exam 1

Spring 2007 Name ______________________________

April 23, 2007

Instructions: Please show your work and clearly indicate your answers. If you use a graphical method to answer a question, sketch your graph with a scale, and give the formula for the function you have graphed. If you use a numerical method, explain what you did. Always specify units!!! Clearly indicate your answers, and be sure your answers make sense. (Highly improbable answers will receive little or no partial credit.) Math grammar counts.
I expect you to exhibit a level of individual academic integrity that is commensurate with being a part of the Honors College. Please acknowledge that integrity by signing the honor statement at the end of the test.

1. The population of Nicaragua was 3.6 million in 1990 and growing at an annual rate of 3.4% per year. Let t be time in years since 1990. [Note: Parts (b)-(f) depend on (a), but not directly on one another. If you can't do (a), ask your instructor for a function to use to answer the remaining questions.]

   (a) Express P as a function of t.

   (b) What is the doubling time of the population?

   (c) How much did the population increase in 2007?

   (d) If $P = f(t)$ is the function you found in (a), find a formula for f^{-1}.

   (e) Explain the practical meaning of f^{-1}. Include units and be careful to say what you mean and mean what you say.

   (f) Find $f^{-1}(15)$ and explain the practical meaning of your computation. Pay attention to units!

2. When a cold yam is put into a hot oven to bake, the temperature of the yam rises. The rate R (in degrees per minute) at which the temperature of the yam rises is governed by Newton's Law of Heating, which says that the rate is proportional to the temperature difference between the oven and the yam. If the oven is at 350°F and the temperature of the yam is H°F,

   (a) Write a formula giving R as a function of H.

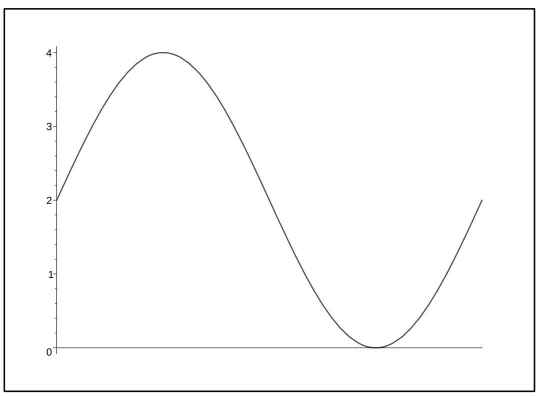
   (b) Sketch a graph of R against H. Label your axes and the horizontal and vertical intercepts of your graph.

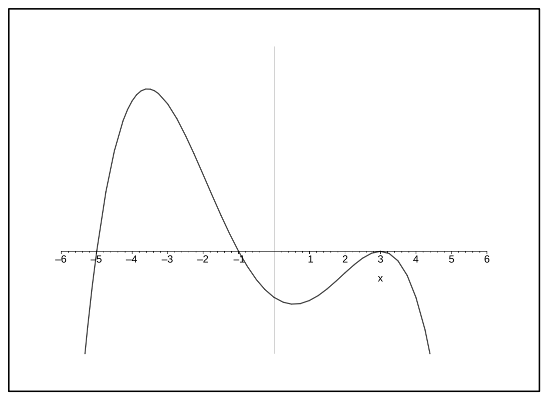
3. The number of hours of daylight varies by season and by longitude. In West Palm Beach, for instance, the number of daylight hours varies between approximately 10.5 hours (December 22), and 13.8 hours (June 21). Find a formula for the approximate number of daylight hours in West Palm Beach as a function of time. Clearly define your independent variable. [Hint: Letting t be days or months after December 22nd or June 21st makes the problem easier!]

4. Values of three functions are given in the table below. Find a formula for each of the functions. Your work must support your answer. Using built-in calculator functions to find formulas will earn you at most 1/4 of the possible credit. [Hint: Each function is linear, exponential, quadratic power, or cubic power.]

t	$f(t)$	$g(t)$	$h(t)$
3	1048	2.2	-4.5
5	524	2.8	-12.5
7	262	3.4	-24.5
9	131	4.0	-40.5

5. Find possible formulas for each of the functions graphed below.





6. **True or False**. Assume $w = f(t)$ is a function for which you know some values (t, w).

(a) _______If a plot of $(\ln t, \ln w)$ is approximately linear, then $w = f(t)$ can be modeled with a linear function.

(b) _______If a plot of $(\ln t, \ln w)$ is approximately linear, then $w = f(t)$ can be modeled with an exponential function.

(c) _______If a plot of $(\ln t, \ln w)$ is approximately linear, then $w = f(t)$ can be modeled with a power function.

(d) _______Every polynomial of odd degree has at least one zero.

7. If possible, choose k so that the following function is continuous on any interval. Briefly explain how you determined your answer.

$$f(x) = \begin{cases} \dfrac{5x^3 - 10x^2}{x-2}, & x \neq 2 \\ k, & x = 2. \end{cases}$$

8. A ball is tossed into the air from a bridge, and its height, y (in feet), above the ground t seconds after it is thrown is given by

$$y = s(t) = -16t^2 + 48t + 64.$$

(a) How tall is the bridge?

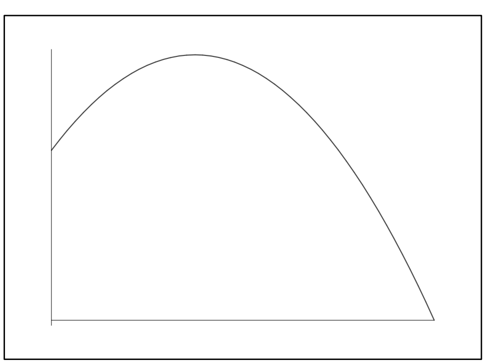
(b) When does the ball hit the ground?

(c) What is the average velocity of the ball on the interval $0 \leq t \leq 2$?

(d) The graph of s is shown. Use your answers to parts (a) and (b) to label the corresponding points on the graph. Next, represent each of the following velocities as the slope of a line (label your lines (i) and (ii), please!):

i. the average velocity of the ball on the interval $0 \leq t \leq 2$, and

ii. the instantaneous velocity of the ball at $t = 2$ seconds.



9. Estimate the limit

$$\lim_{x \to 0^+} x^x,$$

by either graphical or numerical methods. Your work should justify your answer.

10. An ant walking along a crack in the sidewalk is $s(t) = 1/t$ feet from the grass after t seconds, with $t \geq 1$. Use the definition of instantaneous velocity as a limit to find the instantaneous velocity of the ant at $t = 2$ seconds. Math grammar counts!

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Signature

Exam 2

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April 23, 2007

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I expect you to exhibit a level of individual academic integrity that is commensurate with being a part of the Honors College. Please acknowledge that integrity by signing the honor statement at the end of the test.

1. Use the definition of the derivative (as a limit) to find a formula for the derivative of $g(x) = 2+1/x$. Math grammar counts! (You should check your answer using the shortcut rules for derivatives, but reporting the derivative without using the definition to arrive at your answer will earn you only 2 points.)

2. Sketch the graph of a function whose first derivative is everywhere negative and whose second derivative is everywhere positive.

3. The population of a herd of deer is modeled by

$$P(t) = 4000 + 500 \sin\left(2\pi t - \frac{\pi}{2}\right),$$

where t is measured in years from January 1st.

(a) Sketch a graph of $P(t)$ for one year.

(b) Use the graph to decide when in the year the population is at a maximum. When is the population maximum? What is that maximum?

(c) When is the population growing the fastest? *Estimate* the rate the population is changing at this time. Include units! Show your work and clearly indicate/describe how you arrive at your answer.

4. In 1990, the population of Mexico was about 84 million and growing at 2.6% annually, while the population of the US was about 250 million and growing at 0.7% annually. Which population was growing faster in 2006, if we measure growth in people/year? Fully justify your answer.

5. A company's revenue R, in thousands of dollars, from car sales is a function of advertising expenditure a, in thousands of dollars. So $C = f(a)$.

(a) What does the company hope is true about the sign of f'? Explain briefly.

(b) Write a complete sentence interpreting each of the following statements in terms of advertising and car sales.

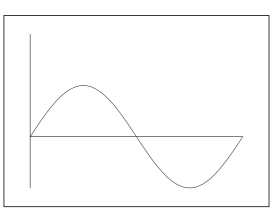
- $f(100) = 500$

- $f'(100) = 0.5$

(c) Suppose the company plans to spend about \$100,000 on advertising. If $f'(100) = 0.5$, should the company spend more or less than \$100,000 on advertising? Explain briefly.

6. (a) Each of the graphs below shows the position of a particle moving along the x-axis as a function of time for $0 \leq t \leq 5$ seconds. The vertical scales of the graphs are the same. During this time interval, which particle has

i. _______Constant velocity?
ii. _______The greatest initial velocity?
iii. _______The greatest average velocity?
iv. _______Zero average velocity?
v. _______Zero acceleration?
vi. _______Positive acceleration throughout?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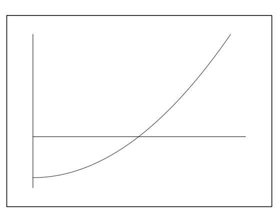
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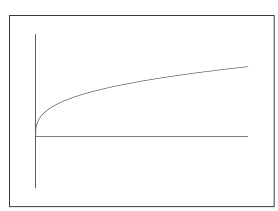
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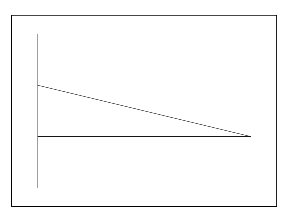
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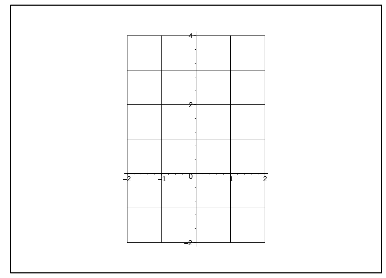
(b) For each of the functions graphed above, sketch the derivative function.

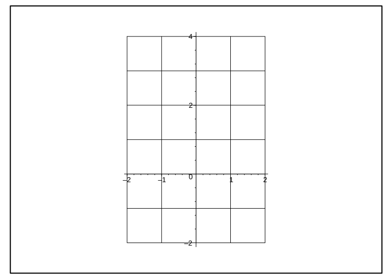
7. Consider the function

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$$

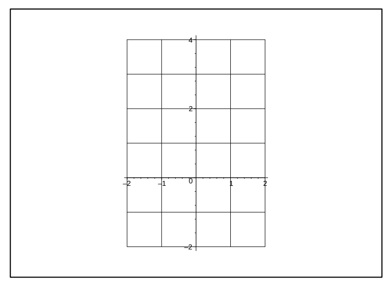
(a) Sketch a graph of this function.



(b) Is f differentiable everywhere? If not, at what point(s) is it not differentiable? Sketch the graph of f' everywhere it exists. [Hint: You should know the derivative of x^2.]



(c) Is the derivative function $f'(x)$ differentiable everywhere? If not, at what point(s) is it not differentiable? Sketch the graph of $f''(x)$ wherever it exists.



8. The period T of a pendulum is given in terms of its length l by

$$T = 2\pi\sqrt{\frac{l}{g}},$$

where g is the acceleration due to gravity (a constant).

(a) Find dT/dl.

(b) What is the sign of dT/dl?

(c) What does dT/dl (including its sign) tell you about pendulums?

9. Suppose f and g are differentiable functions with the values shown below. Find each of the requested values.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	3	4	5	-2
4	1	6	3	0

(a) $h'(2)$ if $h(x) = g(x)/f(x)$

(b) $h'(2)$ if $h(x) = f(g(x))$

(c) $h'(2)$ if $h(x) = \sqrt{g(x)}$

10. Let $y = f(t)$ be the amount of fuel (in pounds) used by a rocket after t minutes of flight.

t	0	0.5	1.0	1.5	2.0	2.5	3.0
$f(t)$	0	35	65	85	101	116	130

(a) Estimate $f'(2)$. What does your answer mean, in practical terms?

(b) Estimate $f''(2)$. What does your answer mean, in practical terms? Show your work so that your reasoning is clear.

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Exam 3

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1. The function $y = A\sin\left(\left(\sqrt{\frac{k}{m}}\right)t\right)$ represents the oscillations of a mass m at the end of a spring. The constant k measures the stiffness of the spring.

   (a) Find a time at which the mass is farthest from its equilibrium position.

   (b) What is the period T of the oscillation?

   (c) Find dT/dm.

   (d) What does the sign of dT/dm tell you, in practical terms?

2. Using the chain rule, develop a formula for $\frac{d}{dx}(\arctan x)$. (Don't just give the derivative of $\arctan x$. Derive it! Start with an identity in which $\arctan x$ is the inside function.)

3. Find the equation of the line tangent to the curve $x^3 + 2xy + y^2 = 4$ at the point $(1, 1)$.

4. (a) Find the tangent line approximation for $\sqrt{1+x}$ near $x = 0$.

(b) Use the tangent line to approximate $\sqrt{1.2}$.

(c) Is your estimate in the previous part an overestimate or an underestimate? Explain your answer graphically.

5. A landscape architect plans to enclose a 3000 square foot rectangular region in a botanical garden. He will use shrubs costing \$25 per foot along three sides and fencing costing \$10 per foot along the fourth side. Find the minimum total cost of the enclosure.

6. Consider the function $f(x) = e^{-x^2}$. Use algebraic methods to answer the following. You may check your work graphically, but you must provide algebraic justification for your answers to receive credit.

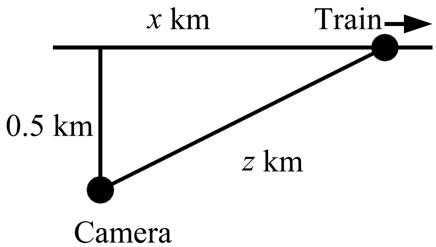
   (a) Find the exact x-values of all critical points.

   (b) Use the derivative to determine the intervals on which f is increasing and on which it is decreasing. Your work should support your answer.

   (c) Find the exact x-values of all inflection points and verify that they are inflection points using a concavity test. Your work should support your answer.

7. The graphs of f, f', and f'' are shown below. Identify which is which.

8. A train is traveling at 0.8 km/min along a long straight track, moving in the direction shown in the figure. A movie camera, 0.5 km away from the track is focused on the train. How fast is the distance from the camera to the train changing when the train is 1 km from the camera? Give units.



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Exam 4

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1. Evaluate the limit algebraically, if possible. If the limit cannot be evaluated algebraically, use another method to evaluate it.

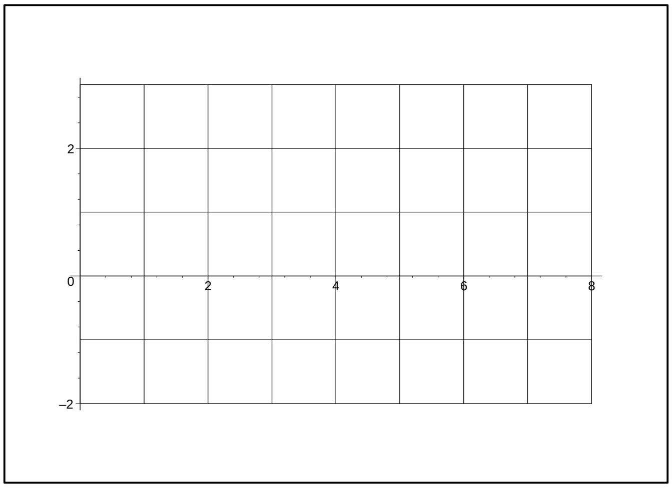
   (a) $\lim_{t \to \pi} \frac{\sin^2 t}{t - \pi}$

   (b) $\lim_{x \to 0^+} \frac{\ln x}{x^2}$

   (c) $\lim_{x \to \infty} \left(\cos \left(\frac{13}{x} \right) \right)^x$

2. A woman drives 10 miles, accelerating uniformly from rest to 60 mph. Graph her velocity versus time. How long does it take for her to reach 60 mph?

3. The graph of $y = f(x)$ is given below. Find the following:



(a) The left hand sum with $n = 3$ subdivisions that estimates $\int_2^8 f(x)\,dx$

(b) The right hand sum with $n = 3$ subdivisions that estimates $\int_2^8 f(x)\,dx$.

(c) The number of subdivisions necessary to estimate $\int_0^4 f(x)\,dx$ with left and right hand sums that differ by at most 0.1.

4. Explain in words, and giving units, what the integral $\int_{2000}^{2004} f(t)\,dt$ represents, where $f(t)$ is the rate at which Florida's population is growing, in millions of people per year, in year t.

5. The rate at which the world's oil is being consumed is continuously increasing. Suppose the rate of oil consumption, in billions of barrels per year, is given by the function $r = f(t)$, where t is measured in years and $t = 0$ is the start of 1990.

(a) Write a definite integral which represents the total quantity of oil used between the start of 1990 and the start of 1995.

(b) Suppose $r = 32e^{0.05t}$. Using a left-hand sum with five subdivisions, find an approximate value for the total quantity of oil used between the stat of 1990 and the start of 1995.

(c) Interpret each of the five terms in the sum from part (b) in terms of oil consumption.

6. Estimate $f(x)$ for $x = 2, 4, 6$, using the given values of $f'(x)$ and the fact that $f(0) = 50$.

x	0	2	4	6
$f'(x)$	17	15	10	2

$f(2) =$

$f(4) =$

$f(6) =$

7. Assume $\int_0^3 f(x)\ dx = 6.$

(a) What is the average value of $f(x)$ on the interval $[0, 3]$?

(b) If $f(x)$ is even, what is $\int_{-3}^3 f(x)\ dx$?

(c) If $f(x)$ is even, what is the average value of $f(x)$ on the interval $[-3, 3]$?

(d) If $f(x)$ is odd, what is $\int_{-3}^3 f(x)\ dx$?

(e) If $f(x)$ is odd, what is the average value of $f(x)$ on the interval $[-3, 3]$?

8. (a) Graph $y = 1 + \sin 2x$ and $y = \cos 2x$ on the interval $[0, \pi]$, then find the exact area between the graph of $y = 1 + \sin 2x$ and $y = \cos 2x$ for $0 \leq x \leq 3\pi/4$.

(b) Given that the area between the graphs of $y = 1+\sin 2x$ and $y = \cos 2x$ for $3\pi/4 \leq x \leq \pi$ is $1-\pi/4$, what is the exact value of $\int_0^\pi (1 + \sin 2x - \cos 2x)\, dx$?

9. A mouse moves back and forth in a straight tunnel, attracted to bits of cheddar cheese alternately introduced to and removed from the ends (right and left) of the tunnel. The graph of the mouse's **velocity** v is given, with positive velocity corresponding to motion toward the right end. Assuming that the mouse starts ($t = 0$) at the center of the tunnel, use the graph to estimate the time(s) at which:

   (a) The mouse changes direction.

   (b) The mouse is moving most rapidly to the right.

   (c) The mouse is farthest to the right of center.

   (d) The mouse is at the center of the tunnel.

   See Exercise 44 in Chapter 5 Review for graph.

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