Exam 1

Fall 2007 Name ______________________________

December 5, 2007

Instructions: Please show your work and clearly indicate your answers. If you use a graphical method to answer a question, sketch your graph with a scale, and give the formula for the function you have graphed. If you use a numerical method, explain what you did. Always specify units!!! Clearly indicate your answers, and be sure your answers make sense. (Highly improbable answers will receive little or no partial credit.) Math grammar counts.
I expect you to exhibit a level of individual academic integrity that is commensurate with being a part of the Honors College. Please acknowledge that integrity by signing the honor statement at the end of the test.

1. (12 points) The median price P of a home in Jupiter rose from \$180,000 in January 2003 to \$300,000 in January of 2005.

   (a) Assume the increase in housing prices was linear. Give an equation for the line representing price P as a function of time t. (Clearly define t!)

   (b) According to your equation in (a), by approximately how much does the median price increase each year?

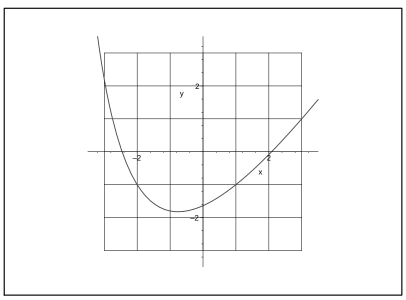
   (c) If instead the housing prices have been rising exponentially, find an equation of the form $P = P_0 a^t$ to represent the median housing price.

   (d) According to your equation in (c), by approximately what percentage does the median price increase each year?

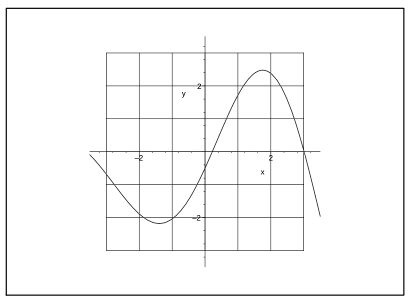
2. (8 points) You drive on the turnpike at a constant speed from Orlando to Miami, a distance of 240 miles. About 170 miles from Orlando, you pass through West Palm Beach. Sketch a graph of your distance from West Palm Beach as a function of time. Your graph should represent your entire trip from Orlando to Miami. Clearly label your axes and the important features of your graph.

3. (8 points) The graphs of $y = f(x)$ and $y = g(x)$ are shown below. Use them to sketch a graph of $y = f(g(x))$. It may be helpful to complete a table of values first!

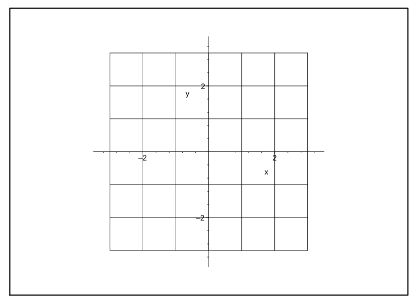
$y = f(x)$



$y = g(x)$



$y = f(g(x))$



4. (8 points) The graph of $y = f(t)$, where $f(t)$ is the number (in millions) of motor vehicles registered in the world in year t, is shown below.

(a) Is f invertible? Explain.

Figure 1.91, page 60

(b) Evaluate $f^{-1}(400)$ and explain the meaning of $f^{-1}(400)$ in practical terms.

5. (10 points) A cup of coffee contains 100 mg of caffeine, which leaves the body at a continuous rate of 17% per hour.

   (a) Write a formula for the amount A of caffeine, in mg, in the body t hours after drinking a cup of coffee.

   (b) Find the half-life of caffeine.

6. (10 points) In the spring, the desert temperature H oscillates daily between 40°F at 5 am and 80°F at 5 pm. Write a possible formula for H in terms of time. Be sure to define your time variable.

7. (8 points) A pomegranate is thrown from ground level straight up into the air at time $t = 0$ with velocity 64 feet per second. Its height at time t seconds is given by $f(t) = -16t^2 + 64t$.

   (a) When does the pomegranate hit the ground?

   (b) When does it reach its highest point?

   (c) What is the maximum height?

8. (10 points) Let $f(x) = \begin{cases} x^2 - 2, & 0 < x < 3 \\ 2, & x = 3 \\ 2x + 1 & 3 < x \end{cases}$

(a) Compute the following limits.

$$\lim_{x \to 3^+} f(x)$$

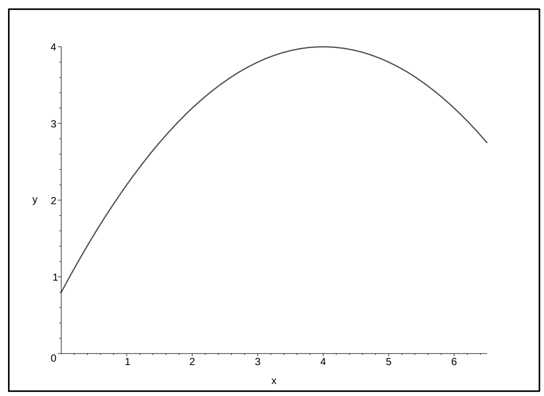
$$\lim_{x \to 3^-} f(x)$$

$$\lim_{x \to 3} f(x)$$

(b) Is f a continuous function? If so, how do you know? If not, at what points is it not continuous, and how could you modify the function to make it continuous?

9. (8 points) The graph of $y = s(t)$ gives the position of a particle at time t. List the following quantities in order, from smallest to largest.

- A, the average velocity between $t = 1$ and $t = 3$
- B, the average velocity between $t = 5$ and $t = 6$
- C, the instantaneous velocity at $t = 1$
- D, the instantaneous velocity at $t = 3$
- E, the instantaneous velocity at $t = 5$
- F, the instantaneous velocity at $t = 6$



10. (8 points) Estimate the derivative of $f(x) = x^x$ at $x = 2$. Your work must support your answer.

11. (10 points) Find the derivative of $g(t) = t^2 + t$ at $t = -1$ algebraically (using the definition of the derivative as a limit). Math grammar counts!

12. (BONUS 5 points) Refer to Question 2 on the test.

    (a) How does your graph illustrate that your speed was constant for the whole trip?

    (b) Assuming your speed was 60 mph, write a formula for your distance from West Palm Beach as a function of time. Be sure to define your variables.

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Signature

Exam 2

Fall 2007 Name ________________________________

December 5, 2007

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1. (8 points) Use the definition of the derivative as a limit to find a formula for the derivative of $f(x) = 1 + 3/x$. Math grammar counts. (Check your answer using the power rule!)

2. (6 points) The graph below shows voltage across an electrical capacitor as a function of time. The current is proportional to the derivative of the voltage; the constant of proportionality is positive. Sketch a graph of the current as a function of time.

   Figure 2.31, page 87

3. (16 points) For some painkillers, the size of the dose D given depends on the weight W of the patient. Thus, for a specific painkiller, we would have $D = f(W)$, where D is measured in milligrams and W is measured in pounds. Be sure to specify units in all responses.

(a) Interpret the statement $f(140) = 120$ in terms of the painkiller.

(b) Interpret the statement $f'(140) = 3$ in terms of the painkiller.

(c) Use the information in the previous parts to estimate $f(145)$.

(d) Explain what the derivative of f^{-1} would tell us in terms of the painkiller.

(e) What is the value of $(f^{-1})'(120)$? What does it tell us in terms of weight and painkiller dose?

4. (8 points) Let $p(t)$ represent the price of a share of stock for ABC corporation at time t. What does each of the following statements tell us about the signs of the first and second derivatives of p?

(a) The price of the stock is rising faster and faster.

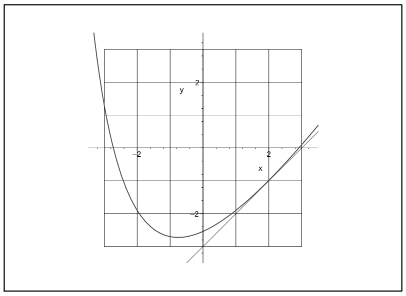
$p'(t)$ ______ $p''(t)$ ______

(b) The price of the stock is close to bottoming out.

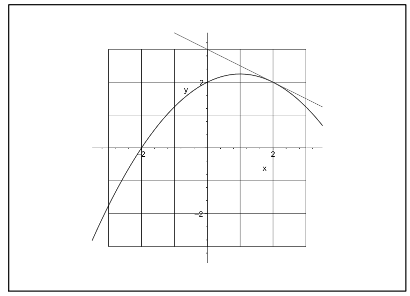
$p'(t)$ ______ $p''(t)$ ______

5. (12 points) The graphs of $y = f(x)$ and $y = g(x)$ are shown below. In each graph, a tangent line is also shown.

$y = f(x)$



$y = g(x)$



Use the graphs to find or estimate the following:

(a) $k'(2)$ if $k(x) = f(x)/g(x)$

(b) $m'(2)$ if $m(x) = f(x)g(x)$

(c) $n'(2)$ if $n(x) = f(g(x))$

6. (8 points) Let $f(t) = \begin{cases} k\cos t, & t < 0 \\ e^{-kt} & t \geq 0 \end{cases}$, where $k > 1$ is a constant.

(a) Is f a continuous function? If so, explain/show how you know. If not, say where it is not continuous and why.

(b) Is f a differentiable function? If so, explain/show how you know. If not, say where it is not differentiable and why.

7. (10 points) A cup of coffee contains 100 mg of caffeine, which leaves the body at a continuous rate of 17% per hour.

   (a) Write a formula for the amount A of caffeine, in mg, in the body t hours after drinking a cup of coffee.

   (b) Find a formula for the derivative of A at time t.

   (c) Find and interpret (giving units!) $A'(1.5)$.

   (d) Is the second derivative of A positive or negative? Explain why your answer makes sense in terms of caffeine. (You do not need to find a formula for the derivative, though you may do so if you like.)

8. (8 points) Use derivatives to determine the interval(s) on which the function $f(x) = x^4 - 4x^3$ is both decreasing and concave up.

9. (6 points) Suppose $f(2) = 3$ and $f'(2) = 4$. Assume f is even. Find:

   (a) $f(-2)$

   (b) $f'(-2)$

10. (6 points) True or False

(a) ________If f is increasing, then f' is also increasing.

(b) ________If f is differentiable, then f is also continuous.

11. (12 points) If you invest p dollars in a bank account at an annual interest rate of $r\%$, then after t years you will have B dollars, where

$$B = P\left(1 + \frac{r}{100}\right)^t.$$

(a) Find dB/dt, assuming P and r are constant. In terms of money, what does dB/dt represent?

(b) Find dB/dr, assuming P and t are constant. In terms of money, what does dB/dr represent?

12. (BONUS 3 points) If $\frac{d}{dx}f(x^2+1) = \frac{2x}{x^2+1}$, what is $f'(x)$?

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Exam 3

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1. (10 points) The depth of the water y in meters in the Bay of Fundy, Canada, is given as a function of time t in hours after midnight by
$$y = 10 + 7.5\cos(0.507t).$$
How quickly is the tide rising or falling at each of the following times? Your work should support your answers. (Please be sure to indicate if the tide is rising or falling.)

   (a) 9 a.m.

   (b) Noon

   (c) 6 p.m.

2. (10 points) Find a formula for the derivative of $\log_{10} x$. Hint: Write an identity in which $\log x$ is the inside function on one side, then take the derivative of both sides. Your work must show that you know how to derive the formula for the derivative, not just write down the answer!

3. (10 points) Find an equation for the tangent line to $x^2 - xy + y^2 = 7$ at the point $(3, 1)$.

4. (10 points)

(a) Find the local linearization of e^{2x} near $x = 0$.

(b) Use your linearization to approximate $e^{0.2}$.

(c) Is your approximation too large or too small? Give a graphical interpretation of your answer.

5. (10 points) A cold yam is placed in a 200°C oven. At time $t = 30$ minutes, the temperature of the yam is 120°C and increasing at an (instantaneous) rate of 2°C per minute. Newton's law of heating says that the temperature of the yam at time t is given by

$$H(t) = 200 - ae^{-bt}.$$

Find a and b.

6. (12 points) Let $f(x) = \dfrac{x}{x^2+36}$. Then $f'(x) = \dfrac{36-x^2}{(x^2+36)^2}$ and $f''(x) = \dfrac{2x(x^2-108)}{(x^2+36)^2}$. Use algebra (not just graphs) to answer the following. Your work must support your answers for credit.

(a) Determine the intervals on which f is increasing and on which it is decreasing.

(b) Identify all critical points of f and classify each as a minimum, a maximum, or neither.

(c) Determine the intervals on which the graph of f is concave up and on which it is concave down.

(d) Find the x-coordinates of the points of inflection.

7. (8 points) The plot below shows the graph of three functions f, f', and f''. Identify which is which.

#39, p 174

8. (15 points) A rectangular swimming pool is to be built with an area of 1800 square feet. The owner wants 5-foot wide decks along either side and 10 foot wide decks at the two ends. Find the dimensions of the smallest piece of property on which the pool and deck can be built satisfying these conditions.

9. (15 points) A chemical storage tank is in the shape of an inverted cone with depth 12 meters and top radius 5 meters. When the depth of the chemical in the tank is 1 meter, the level is falling at 0.1 meters per minute. How fast is the volume of chemical changing? [The volume of a cone is $\frac{1}{3}\pi r^2 h$.]

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Exam 4

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1. (10 points) Determine whether or not the following limits exist, and evaluate those that do. Your work must support your answer!

   (a) $\lim_{x \to 1} \frac{\ln x}{x^2 - 1}$

   (b) $\lim_{x \to 0} \frac{\sin^2 x - xe^{2x}}{x}$

2. (4 points) Let $f(t)$ be the rate at which the US population is growing, in millions of people per year, in year t. Explain in words what $\int_{2000}^{2007} f(t)\,dt$ represents, including units.

3. (10 points) Roger runs a marathon. His friend Jeff rides behind him on a bicycle and clocks his speed every 15 minutes. Roger starts out strong, but after an hour and a half he is so exhausted that he has to stop. Jeff's data frollow:

Time since start (min)	0	15	30	45	60	75	90
Speed (mph)	10	9	8	8	7	6	0

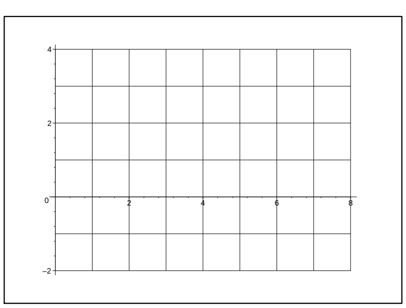
(a) Assuming that Roger's speed is never increasing, give upper and lower estimates for the distance Roger ran during the hour and a half.

(b) Give your best estimate of the total distance Roger ran. What is the maximum possible error in your estimate?

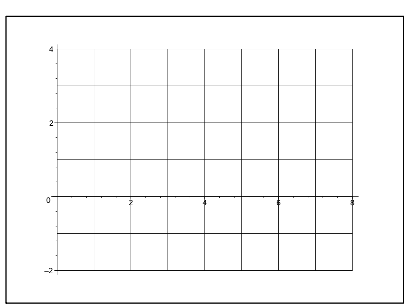
(c) How many (equally-spaced) measurements would Jeff have needed to make in order for you to estimate the total distance Roger ran with error at most 0.1 mile?

4. (12 points) The graph of $y = f(x)$ is shown in the figures. All important values are integers. Find the requested estimates of $\int_1^7 f(x)\,dx$ and show how each estimate can be visualized on the graph of $y = f(x)$.

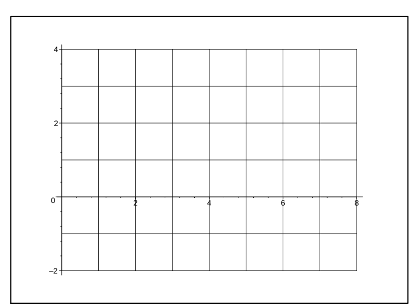
(a) LHS(2)



(b) LHS(6)



(c) RHS(3)



[f is continuous and contains the points: $(0,2), (1,0), (2,-2), (3,-1), (4,2), (5,3), (6,4), (7,3), (8,2)$]

5. (8 points)

   (a) Find (or estimate) the average value of $f(t) = \sqrt{25 - t^2}$ on the interval $[-5, 5]$.

   (b) How can you tell that this average value is greater than 2.5 without doing any calculations?

6. (16 points) The graph of $y = f(x)$ is shown. Find the following: Figure 5.70, page 273

   (a) the total area between the graph of $y = f(x)$ and the x-axis for $0 \le x \le 5$

   (b) $\int_0^5 f(x)\, dx$

   (c) $\int_3^5 f(x)\, dx$

   (d) $\int_3^0 f(x)\, dx$

   (e) $\int_0^5 |f(x)|\, dx$

   (f) A graph of an antiderivative $F(x)$ of $f(x)$ satisfying $F(0) = 2$. Label each critical point of F with its coordinates.

7. (4 points) Ice is forming on a pond at a rate given by $\frac{dy}{dt} = 0.12\sqrt{t}$, where y is the thickness of the ice, in inches, t hours since the ice started forming. Find y as a function of t.

8. (16 points) A bicyclist is pedaling along a straight road for one hour with a velocity v shown below. She starts out 5 km from a lake. Positive velocities take her toward the lake and negative velocities take her away from the lake.

Figure 5.74, page 274

(a) Does the cyclist ever turn around?
If so, at what time(s)?

(b) When is the cyclist going the fastest?
How fast is show going then?
Toward the lake or away?

(c) When is the cyclist closest to the lake? Approximately how close to the lake does she get?

(d) When is the cyclist farthest to the lake? Approximately how far from the lake is she then?

9. (10 points) A cat, walking along the window ledge of a New York apartment, knocks off a flower pot, which falls to the street 200 feet below. How fast is the flower pot traveling when it hits the street? Give your answer both in feet per second and also in mph, given that 1 ft/sec = 15/22 mph.

10. (BONUS 4 points) Sketch the graph of a function f (you do not need to give a formula for f) on an interval $[a, b]$ with the property that for $n = 2$ subdivisions,

$$\int_a^b f(x)\,dx < \text{Left-hand sum} < \text{Right-hand sum}.$$

11. (10 points) Consider the accumulator function $F(x) = \int_1^x \frac{1}{\arctan t}\, dt$.

(a) Find $F'(x)$.

(b) Is $F(x)$ increasing or decreasing on $[1, \infty)$? Explain briefly. What can you say about the concavity of F on $[1, \infty)$? Explain briefly.

(c) Find $\frac{d}{dx} F(x^2)$.

(d) Find or approximate $F(1)$ and $F(3)$. Use your results from this part, as well as (b) to sketch a graph of $F(x)$ on $[1, 3]$.)

12. (BONUS 6 points)

(a) For what values of C and n (if any) is $y = Cx^n$ a solution to the differential equation

$$x\frac{dy}{dx} - 3y = 0?$$

(b) If the solution satisfies $y(2) = 40$, what more (if anything) can you say about C and n?

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Signature