**Print name:** _____ **Spring 2007, Test 1**

**Calc 1** **Dr. Eugene Belogay**

Justify all answers and show all work — absurd answers or answers with no work get no credit. Display all computing steps. Write clearly — neatness counts. Scratch all the work that you don't want graded. Write each answer in the provided box. Use the back of the page for extra space. I will ignore anything I cannot read (or find). Unless otherwise noted [in brackets], each problems is worth 15 points (the total is 100). Closed books, no notes. Calculators are allowed.

*"I upheld the principles of the Honor Code.* *Signature: X*_____*"*

**1.** [20] Write a possible formula for each function described or plotted below:

**(a)** having a vertical asymptote at $x = 1$ and horizontal at $y = 2$.

**(b)** p.15, prob.20,

**(c)** p.35, prob.27

**(d)** p.42, prob.12

**Extra** [3] p40, example 3

**2.** [15] Limits.

**(a)** Use your calculator to estimate $L = \lim_{h \to 0} \dfrac{(-1+h)^2 - 1}{h}$. $< L <$

[*Hint: use both positive and negative values for $h$, such as $\pm 0.1, \pm 0.01$*]

**(b)** Use algebra to compute the exact value of $L$.

**(c)** Fill in the blanks in the following statement:

The above limit is equal to $f'(\quad)$, where $f(x) =$ .

**3.** [15] Some of the values of the linear function $f$ and the exponential function $g$ are tabulated below. Fill in the blanks with exact values (not estimates).

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f(x)$ | 10 | | 20 | | |
| $g(x)$ | 10 | | 20 | | |

**4.** [10] Sketch a possible graph of distance versus time in each case, so that:

**(a)** the average velocity from $t = 0$ to $t = 3$ is the same as the instantaneous velocity at $t = 2$, but the velocity is not constant.

**(b)** the plot of distance from the dorm matches the following story:
I had just left the dorm on my way to the beach when I realized I had forgotten my driver's license in my room, so I turned around and went back to pick it up. After an hour at the beach, I left for home, but stopped at Publix to get a sandwich.

**5.** [8] Let $p$ be the price of an item and $q = f(p)$ be the quantity of the items that can be sold at that price. What does each of the following quantity represent? Explain in English and include units.

**(a)** $f(20)$

**(b)** $f^{-1}(100)$

**6.** [12] The water bill for each household in a small town includes a fixed flat fee plus the cost of each cubic foot of water used. The Abbots were charged \$90 for 1000 cubic feet, while the Browns were charged \$105 for 1600 cubic feet.

**(a)** How much does each cubic foot of water cost?

**(b)** Write an equation for the water bill.

**(c)** How much water can one buy for \$130?

**7.** [20] The amount $Q$ of radioactive iodine-131 in the thyroid gland of a patient is given by the formula $Q(t) = 20(0.5)^{t/8}$, where the time $t$ is in days, and $Q$ is in milligrams.

**(a)** How much iodine-131 was given to the patient?

**(b)** How much iodine-131 remained after 16 days?

**(c)** What is the half-life of iodine-131?

**(d)** What percent of iodine-131 decays each day?

**(e)** How long until only 0.1% of the original iodine-131 remain?

**Print name:** _____ **Spring 2007, Test 2**

**Calc 1** **Dr. Eugene Belogay**

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**1.** The Power Rule states that the derivative of $f(x) = 1/x^2$ is $f'(x) = -2x^{-3}$. Derive this formula by computing the limit in the definition of $f'(x)$.

**2.** Use the table data

| $x$ | 0.2 | 0.4 | 0.6 |
|---|---|---|---|
| $f(x)$ | 3.5 | 3.5 | 3.1 |

in order to:

**(a)** estimate $f'(0.3)$; $f'(0.3) \approx$

**(b)** estimate $f'(0.5)$; $f'(0.5) \approx$

**(c)** estimate $f''(0.4)$; $f''(0.4) \approx$

**(d)** determine the concavity of $f$ at $x = 0.4$.

**3.** Suppose that a company's revenue from sales, $R$ (in thousands of dollars), is a function of the amount spent on advertising, $A$ (also in thousands of dollars), i.e., $R = f(A)$.

**(a)** What is the sign of $f'(A)$? Why?

**(b)** Interpret the statement $f'(60) = 1.5$.

**(c)** Knowing that $f'(100) = 0.8$, advise the managers whether they should increase the advertising expenses to \$110,000. Explain your reasons.

**4.** Assume $r(2) = 4$, $r'(2) = 2$; $s(1) = 3$, $s(2) = 1$, $s(4) = 2$, $s'(1) = -2$, $s'(2) = 3$, $s'(4) = 0$. Compute $H'(2)$,

**(a)** if $H(x) = \dfrac{s(x)}{r(x)}$. $H'(2) =$

**(b)** if $H(x) = \sqrt{s(x)}$. $H'(2) =$

**(c)** if $H(x) = s(r(x))$. $H'(2) =$

**(d)** if $H(x) = 2^{r(x)}$. $H'(2) =$

**5.** Which of the following statements are always true? Circle the correct responses.

**(a)** The formula (distance traveled) = (average velocity) (time elapsed) is valid for every moving object. True False

**(b)** By definition, the instantaneous velocity is a difference quotient. True False

**(c)** Linear functions have constant derivatives. True False

**(d)** If $g(x)$ is a vertical shift of $f(x)$, then $g'(x) = f'(x)$. True False

**(e)** $(fg)'' = f''g + fg''$. True False

**(f)** The derivative of every polynomial is a polynomial. True False

**(g)** The derivative of $\tan x$ is periodic. True False

**(h)** If $f(x)$ is increasing, then $f(x)'$ is increasing. True False

**(i)** If $f'(x)$ is increasing, then $f(x)$ is increasing. True False

**(j)** If $f''(x) > 0$, then $f'(x)$ is increasing. True False

**Extra** [4] Sketch a possible graph of a function $f(x)$, such that $f(x) > 0$, $f'(x) > 0$, and $f''(x) < 0$ for $-5 < x < 5$. Can such a function be defined for all $x$? Why?

**6.** (2.2:13) Use the figure to fill in the blanks about the behavior of $f(x)$ at the point $B$:

**(a)** $f(\_\_) = \_\_$

**(b)** $f'(\_\_) = \_\_\_\_$

**7.** Each column below contains the graphs of $f, f', f''$. Which is which? Label them appropriately and explain your decision.

**8.** Four particles are moving along the $x$-axis according to the graphs on the right (See Fig 2.61 on p.105). Which particle has:

**(a)** constant velocity throughout?

**(b)** largest average velocity?

**(c)** largest initial velocity?

**(d)** zero average velocity?

**(e)** zero acceleration?

**(f)** only positive acceleration throughout?

**Print name:** _____ **Spring 2007, Test 3**

**Calc 1** **Dr. Eugene Belogay**

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**1.** Write parametric equations (with parameter $t$) for:

**(a)** the circle of radius 2 centered at (2,0)

$x =$ , $y =$

**(b)** the straight line through the points (1,2) and (3,5)

$x =$ , $y =$

**(c)** the parabola $y = x^2$

$x =$ , $y =$

**(d)** the tangent line to the parabola at (1,1)

$x =$ , $y =$

**2.** Find the slope and the equation of the line tangent to the graph of $x \ln y + y^3 = \ln x$ at the point $(e, 1)$.

slope =

equation: $y =$

**Extra** [2] Using the equation of the tangent line, approximate the $y$-value for $x = e + 0.1$.

$y(e + 0.1) \approx$

**3.** Patrick makes skateboards in his garage and sells them for \$50 each. He estimates that his total cost (expense) for building $x$ boards in a month is given (in dollars) by

$$E(x) = .03x^3 - 2.25x^2 + 58.64x + 300.$$

Assuming that Patrick can sell all the skateboards he makes, what is his maximum monthly profit (= price – cost)? Justify your answer.

**4.** The graph of $f'$ (**not** $f$) is given below. At which of the marked points $0, x_1, \ldots, x_5$ (mark **all** that apply) is the original function $f$:

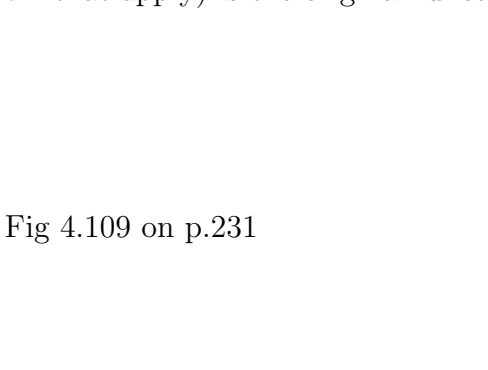

Fig 4.109 on p.231

**(a)** At a local max? $x =$

**(b)** At a local min? $x =$

**(c)** Climbing the fastest? $x =$

**(d)** At an inflection point? $x =$

**5.** Consider $g(x) = xe^{-x}$ for $x \geq 0$. Where is the graph of the function increasing? Concave down? Report exact $x$-values using derivatives.

increasing:

conc. down:

**Extra** [2] Compute $\lim_{x \to 0} g(x)$ and $\lim_{x \to \infty} g(x)$ .

**6.** A Japanese beetle infestation is spreading from the center of a small town. Since beetles fly in all directions, we assume the region they cover is circular. Suppose the radius of this circular region is increasing at a rate of 1.5 miles per year. How fast is the infested area growing when its radius is 6 miles? (Include units.)

$\frac{dA}{dt} =$

**7.** [20] A 13-foot ladder resting on horizontal ground is leaning against a vertical wall when its base starts to slide away from the wall at a rate of 6 feet per second.

**(a)** How fast is the top sliding down when the base is 5 ft from the wall? $\frac{dy}{dt} =$

**(b)** What is the maximal area of the triangle formed by the ladder, the wall, and the ground?

$A_{\max} =$

**8.** A landscape architect plans to enclose a rectangular area of 300 square feet. She wants to plant shrubs at \$20 per foot along three sides and fence the fourth side at \$10 per foot.

Find the minimal cost and the optimal sizes of the area.

opt. size = ft × ft

min. cost = \$

**9.** Consider the function $f(x) = \sqrt{1 + \sin x}$ for $x$ near 0.

**(a)** Compute $f(0)$ and $f'(0)$.

$f(0) =$ , $f'(0) =$

**(b)** Write the tangent line approximation of $f$ near $x = 0$.

$f(x) \approx$

**(c)** Using your results above, approximate $\sqrt{1 + \sin(0.04)}$ **without** using a calculator.

$\sqrt{1 + \sin(0.04)} \approx$

**Extra** [2] Write the equation of the tangent line to the graph of $f$ at $x = 0$.

$y =$

**Print name:** _____ **Spring 2007, Test 4**

**Calc 1** **Dr. Eugene Belogay**

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**1.** Pollutants in industrial gas are removed by scrubbers, which become less efficient over time. The pollutant escape rate is measured each month (in tons/month) and displayed in the following table.

| Time (months) | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Escape rate (tons/mo.) | 5 | 7 | 8 | 10 | 13 | 16 | 20 |

**(a)** Compute upper and lower estimates for the total amount of pollutants that escaped during the six months. Include units.

upper est.=

lower est.=

**(b)** How often should the measurements be made in order to find lower and upper estimates that differ by at most one ton?

every

**2.** Compute

**(a)** $\int (t+5)^2\,dt$

**(b)** $\int 2^x\,dx$

**(c)** $\int \sin(2x)\,dx$

**(d)** $\int \frac{x+1}{x}\,dx$

**(e)** $F(x)$, if $F(0)=4$ and $F'(x)=\sqrt{x}$.

**Extra** [3] $\int_{-\pi}^{\pi} t^3 \cos x\,dx$

**3.** The graph of $P'$ (not $P$) is shown below. Fill in the table of values for $P(t)$ given that $P(0) = 2$, then carefully sketch the graph of $P$.

6.1:8 Fig. 6.11 p. 285

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $P(t)$ | 2 | | | | | |

**4.** Let $F'(x) = f(x)$; the graph of $f$ is given below. Mark the following quantities on the graph (it should be clear if each represents length, area, or slope).

5.4:36-39 Fig 5.66 p.272

**(A)** $f(b) - f(a)$

**(B)** $\frac{f(b)-f(a)}{b-a}$

**(C)** $F(b) - F(a)$

**(D)** $\frac{F(b)-F(a)}{b-a}$

**Extra** [3] Describe (D) in words

**5.** [10] A car speeds up from 0 to 60 mph for 5 miles with constant acceleration. Find the average speed of the car and the time it took to travel the 5 miles. Include units!

$v_{\text{avg}} =$

time =

**6.** Daily sales at Patrick's skateboards are increasing, but at a decreasing rate. Let $f(t)$ represent the daily sales (in dollars per day) of skateboards on the $t$-th day of the month.

**(a)** Which is greater: $\frac{f(0)+f(30)}{2}$ or the average value of $f$ on $[0, 30]$? Use a picture to explain your reasons.

**(b)** Assuming that the Riemann sums below are estimates for the definite integral $\int_0^{30} f(t)\,dt$, put the following numbers in increasing order: LHS(5), RHS(5), RHS(10), $\int_0^{30} f(t)\,dt$.

**7.** The Montgolfier brothers were eighteenth-century pioneers in the field of hot-air ballooning. Had they had the appropriate instruments, they might have left us a record of one of their early experiments, like that shown below. The graph shows their vertical velocity $v$ (with upward as positive).

(Graph from Chapter 3 Review, Problem 22)

**(a)** Over what time intervals was the balloon decelerating?

**(b)** What was the greatest upward velocity?

**(c)** When did the descent start?

**(d)** What was the greatest altitude the balloonists achieved? When?

**(e)** This particular flight ended on top of a hill. How do you know that it did, and what was the height of the hill above the starting point?