

Creating Dynamic Problem Solvers While Learning Part-Whole Concepts: Young Children Using Manipulatives for Mathematics Learning

At a time when Science, Technology, Engineering and Mathematics (STEM) professions are in great demand, the question as to how to cultivate learners' interests in these fields becomes more urgent. This paper presents valuable guidance in building positive mathematics learning experiences among students of all ages. Using effective, enjoyable, and real-life activities, it presents an innovative, research-based, and field-tested model approach to teaching Part-Whole concepts to young children. Building upon extended action research, this project reveals how using a Rekenrek manipulative and Think Aloud strategies, mathematics teachers cultivate not only number sense and arithmetic skills, but also the internalization of essential pre-Algebra concepts. Instructional activities, illustrations, as well as facilitation strategies are described in detail to provide specific examples for instructors to explore. This article describes the research foundation of the models, and a brief theoretical background and history of these approaches. However, the main foci of this paper are the several real-life applications for the math teaching and learning process which emerged. The final section of the article returns to a discussion of the foundational educational principles which undergird the approach and outcomes of these multisensory and manipulative math instructional approaches.

Introduction

“Teacher, there is another way to solve this problem!

I have an idea: what if we do it *this* way?”

Such comments are common in classrooms of educators who strive to cultivate a love of Algebraic thinking through what appear to be games and fun activities. These teachers facilitate the development of young learners' Algebraic thinking through math manipulatives including real-life scenarios and toys. With this approach, students experience a progression from real-life scenarios to the development of problem-solving strategies. In addition, they not only grasp and internalize these essential strategies, but they also articulate, document, and generalize them.

At a major metropolitan college in the New York state, in-service teachers and teacher researchers continue to be more invested in teaching math because they see their young students taking ownership of math learning and building Algebraic concepts early in life. A major

focus of the math education courses is that they integrate math manipulatives, student think-aloud(s), modeling, and math concepts. This integrated, constructivist, and innovative approach was developed over several years of careful action research with teachers and classrooms in nearby public and private schools (Lyublinskaya, & Kerekes, 2008).

The results are that students and teachers alike enjoy and build greater confidence in math, in part because everyone experiences more success. That is, the students are motivated to learn, and teachers are more motivated to teach math. In order to encourage other instructors and teacher education programs to consider these means of facilitating greater motivation and confidence in math learning and teaching, this article presents a vital instructional segment of these successful teacher education classes and research projects.

Specifically, this paper presents the research-based origins of these classroom strategies which facilitate student mathematic learning and develop Algebraic thinking among

young learners using the Rekenrek (Treffers, 1997, 1998) and the mathematical concept Part-Whole Relationships. The first part of the article reviews the research model, emergent research questions, related literature, and approaches, which were developed and tested. The extensive and varied collected data illustrate the results of these action research efforts. The integrated presentation of several real-life mathematical teaching and learning applications provide an overview and practical instructions for classroom use. In this manner, we blend research, theory and practice to inform motivating, research-based math education of young children.

The Need: Background and Outline of Math Education Strategies

In understanding this constructivist approach for foundational Algebra thinking, several unfamiliar yet vital concepts emerge. Two of these critical concepts are that young students (1) need to develop basic Algebra thinking to understand addition and subtraction fully, and (2) once they understand these mathematical concepts, they will be able to grasp Algebraic thinking more easily (Fosnot & Dolk, 2001; Kamii & Dominick, 1998).

Overview of the Literature: Foundational Points for Understanding and Using the Emergent Instructional Model

This section of the article provides an overview of critical concepts from math education and the literature which are the foundational elements for the research and discussion presented herein. The nature of the emergent action research guides us to include the following major topics: Foundations for Algebraic Thinking, Rekenrek, and Think-Aloud methods.

Strong Foundation for Algebraic Thinking

In its entirety, the approach researched in this article develops a foundation for students' Algebraic thinking without needing any Algebra specific language. Instead of imposing others' solutions upon them, students internalize number sense through active learning with manipulatives, solving real-life scenarios, and exploring many

different combinations of numbers for solutions. Research conducted by Walcott, Mohr, and Kastberg (2009) also revealed the benefits of using three-dimensional manipulatives in building mathematical sense-making.

Several critical mathematical developmental achievements of young children include counting by fives, ones, and tens up to 20 (Crane, 2013; Lembke & Foegen, 2009; NCTM, 2000). In this instructional model using real-life scenarios, the Think-Aloud process and the Rekenrek, students internalize these understandings, instead of learning these skills by rote. This achievement provides the development of an order of arithmetic skills beyond rote recitation. Multiple number combinations, arithmetic functions and personal explanations of their choices demonstrate the depth of this learning and the difference.

The Rekenrek

One of the learning manipulatives used in this study is the Rekenrek. The Rekenrek (See Figure 1) is a valuable tool to use as a manipulative in building additional mathematical understanding for young students, typically prekindergarten to 2nd grade (Tournaki, Bae, & Kerekes, 2008; Treffers, 1997, 1998). Similar to an abacus (found throughout ancient China, Egypt, Rome, etc), it was developed in the Netherlands but has a major notable difference. The Rekenrek has two strings of 10 beads. On each string, five beads are red and five are white. This simple characteristic sets the Rekenrek distinctly apart from other abacus-related manipulatives found in most schools today. The details of the impact of this structural difference on math learning will be explained in the course of this paper.

In the math education and classroom learning contexts, the Rekenrek scaffolds the benefits of using multiple manipulatives into one simple device. Otherwise teachers often would have to use a number line, counters, and/or Base-10 blocks to accomplish similar outcomes which are gained from using the Rekenrek alone (Tournaki, Bae, & Kerekes, 2008).



Figure 1. The Rekenrek

The Think Aloud Method: From Narration to Analysis

The Think-Aloud method is an essential meta-cognition and communication strategy for this instructional approach. With Think-Aloud, students articulate and narrate their thinking process while engaged in problem solving. By providing students with opportunities to practice Think-Aloud, they learn how to narrate better, eventually articulate, ultimately being able to analyze and discuss their thought processes (van Someren, Barnard, & Sandberg, 1994).

Synergies

In the case of the Rekenrek activity used in this research, the Think Aloud method affords many benefits. The combination of this manipulative and instructional approach integrates several elements less frequently practiced and often isolated from one another: visual perception, motor skills, and mathematical learning (Tournaki, Bae, & Kerekes, 2008). The discussion of the findings of the research will reveal the specific benefits of this synergy for mathematical teaching and learning

Research Methodology

Method

The methodological approach for this study is action research. Action research is especially well suited for solving instructional needs which arise in classrooms, allowing educators to pose solutions, and testing those potential solutions (Creswell, 2014; Hinchey, 2008). A tremendous benefit of action research is that it is a close to the ground approach. This means that it affords dynamic revision throughout the research project

and concomitant instructional intervention. Critical to the study at hand, significant strengths of action research as a research method is its formative, data-driven decision making focus, and teacher-as-research orientation (Hinchey, 2008; Izzo, 2006).

Conducting action research in a teacher education course in math instruction serves double duty. On the one hand, these educators become more adept in math instruction practice, and on the other hand, they also experience the relevance of research. Engaging in such inquiry also cultivates teacher research perspectives (Izzo, 2006; Nevarez La Torre, 2010). Moving the locus of power for conducting action research to the classroom teacher provides a stronger orientation and foundation for instructional decision making, and also a much greater comprehension of research issues, questions, and procedures (Hinchey, 2008; Stringer, 2014).

Although not broadly generalizable, an action research study affords a model for educators with similar settings and participants to learn and inform their work. Additionally, action research studies provide examples for educators to examine before they attempt to use new methods (Stringer, 2014). Reading accounts of action research in mathematics teaching and learning can provide classroom teachers with many more strategies and models for instruction, data driven decision making, and research inquiry.

Although teacher education is a widely studied field, the use of this model of mathematical instruction as a teacher action research project is less familiar and documented. Therefore, the outcomes, patterns and recommendations of this research can guide future study (Stringer, 2014).

Data

Several different forms of data comprise the body of research for this study. Both teacher candidate reflections and the professor's field notes and observations were documented and analyzed for this article. Moreover, classroom data included classroom presentations, prek-4 student artifacts, photographs, video clips, teacher researcher content focus/discussion groups, and teacher researcher journals.

Analysis of these data was conducted through constant comparison of emergent themes in order to develop grounded theory (Charmaz, 2014; Creswell, 2014; Glaser & Strauss, 1967). The narrative accounts were repeatedly read and analyzed for common themes; these themes were identified and the texts coded. This process was continued until saturation (when no more themes were evident). In addition, in our analysis we developed several data displays of observations, frequency counts, themes, observations and patterns in table format, flow charts, star charts and visual models (Creswell, 2014). At this point, the authors examined the results and the case to determine evidence of a grounded model or theory having emerged (Charmaz, 2014).

Participants

The direct participants in this study included one professor of math education, four teacher researchers who were enrolled in the graduate course, and the prekindergarten – 4th grade students enrolled in each of their classes. The teachers were already classroom instructors and had returned to graduate school to earn their Master's degrees. They were enrolled in their Master's seminar and the action research was their capstone research for their graduate study.

A second professor of education collaborated on analysis of the research findings and co-authored the article manuscript. An external research collaborator provided valuable outside perspective of the process and findings. Moreover, our data analysis discussions of the project have also generated new understanding of the instructional process and outcomes.

A powerful opportunity is afforded in examining the collaborative efforts of a professor with her teacher-researchers, together focused on the needs of their P-4 math classroom instruction as action research. Mentoring into professional perspectives and practice of reflection, action research, accountability and collaboration can be synthesized in such learning communities (Izzo, 2006).

Research Framework

The entire learning experience and research project extended from September to May 2009 (nine months). During this time several significant stages of the project unfolded and led from one to the next. Data analysis and manuscript development has spanned another three months of intense collaboration, model development and vetting.

Figure 2 illustrates these stages of the research and professional learning process. The figure has been created as part of the analysis and manuscript development process. While it will be helpful to use with future teacher researchers, it is important to know that the participants in this study did not have the benefit of seeing this comprehensive diagram. Instead, they experienced the action research process one step at a time, and reflected on it in the same sequence. After the fact, they were able to reflect on the entire process.

In Figure 2, the larger outer circle is the entire nine-month project. This cycle includes college classroom instruction in research methods and demonstrations of the instructional strategies. This foundation is built upon by the classroom observations and the teacher researchers' identification of action research questions in their P-4 classrooms. As the professors and teacher researchers reconvene, they determine to identify new instructional strategies. Stage 6 is the implementation of these strategies in the P-4 classroom. In Stages 7 and 8, one sees the truly regenerating stages as instructors continue to develop new strategies and recognize more needs which can be addressed. This phase of the cycle culminates the power of action research to provide a problem solving, data driven basis of effective, differentiated math instruction.

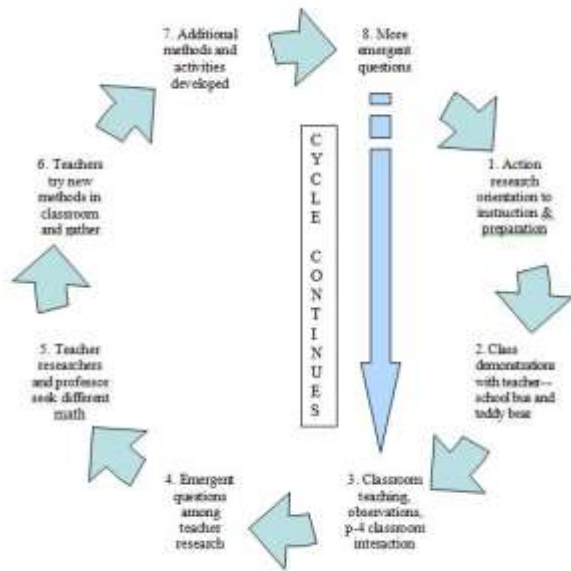


Figure 2. Action Research Cycle for Teachers' Professional Learning (For the reader's convenience, Figure 2 is magnified in Appendix 1)

Nonetheless, this cycle is a continuing one and it is represented by the arrow which dissects the circle from Stage 8 to Stage 3. This smaller sequence represents a continuing subset or sub-cycle as it were. This subset of the entire cycle is Stages 3 through 8 and they represent the continuous process of instructional improvement and action research in which classroom teachers would engage.

As illustrated in Figure 2, this action research cycle is one which incorporates several steps, some used only during the teacher researchers' formal class enrollment and others (Stages 3 through 8 cycles) which can be continued. This cycle can be used by the professional educators as they continue to develop and refine their math education pedagogy and practice from an action research orientation.

Action Research Question

Essential to the practice of action research is the contextual origin of the research question (Hinchey, 2003). In this research study, the teacher researchers observed some of their P-4 students having difficulty learning and understanding the

traditional carry over method of addition and subtraction of larger numbers. There was a teacher research focus group which met weekly. Therefore, this difficulty was posed to them as a question and they discussed possible instructional solutions. Together the professor and teacher researchers decided to investigate. "What other mathematical teaching and learning methods might be used to overcome the difficulty?" This was the process through which the research question for this study emerged.

The collaborative group's investigation for an alternate instructional approach revealed the Rekenrek (Treffers, 1997, 1998) of the Netherlands as a viable option to try in their classrooms. The next section of this article reveals the instructional plan generated by the collaborative action research group. Moreover, it reveals the strategies used to cultivate learning and deliver each lesson.

Proposed Instructional Plan, Strategies and Findings

Instructional Plan

Six phases of math instruction strategies were developed, trialed and implemented by the teacher researchers. These phases are illustrated as related to the young students' math developmental understanding in Appendix 1, Sequence of Young Students' Developing Understanding Part-Whole Relationships Using Manipulatives and Real-Life Activities. The six phases and related instructional strategies are described and discussed in this section for two major reasons. Firstly, this emergent approach arises from action research inquiry. Secondly, the strategies need to be described in order to understand the research findings in context. In order to sustain connections among strategies and findings, in each section the instructional approach will be described, followed by the related findings.

Instructional Strategies and Findings

In the first phase of this sequence of activities, the teacher uses toys as math manipulatives, constructing stories and engaging students in internalizing number sense and

problem solving. The following example reveals how a teacher may facilitate children's hands-on discoveries of number sense and problem solving in this manner.

Real-life Context Addition and Subtraction: Toys and Bus

Materials needed: Several sets of miniature plastic teddy bears, Matchbox™ type bus, bus stop flag and stand, and map.

Overview of lesson: The teacher models the following scenario with the toy bears and the school bus (adapted from Fosnot and Dolk, 2001).

Teacher's script:

"The two teddy bears want to go to school and they climb on the bus." (The teddy bears sit on top of the bus to ride it.)

"At the next bus stop, another three teddy bears get on the bus."

Ask the students, "How many teddy bears do we have on the bus all together?"

After the students respond, "How do you know that?" or "How did you come to that answer?"

"One of the teddy bears has a stomach ache and wants to go home. So the bus driver calls his parent and let's the teddy bear off the bus at the next stop where her parent is waiting."

Ask the students, "How many teddy bears do we have on the bus now?"

After the students respond, "How do you know that?" or "How did you come to that answer?"

Depending on level of student understanding, teacher might ask the class, "If we had ten seats to start, how many are left?"

Develop Part-Whole Relationship with the Rekenrek

In the second phase of this sequence of activities, teachers use the Rekenrek with the students as a vehicle for play *and* learning. They construct stories and engage students in internalizing number sense and problem solving. The Rekenrek Doubling Activity example reveals how a teacher may facilitate children's hands-on discoveries of number sense and problem solving in this manner

When introducing the Rekenrek, teachers need to explain the structure/construction of the manipulative by asking questions of the students. As the teacher introduces real-life scenarios, they model them on the Rekenrek to explain and develop additional number sense and math strategies.

Teachers model the activity and facilitate students' expression and development of their understanding, using real-life stories, this time with the Rekenrek. Students explore, practice and learn to use this manipulative to express their strategies, while the teacher draws out their strategy and solutions through Think-Aloud.

Learning about Implementing Think-Aloud with the Rekenrek.

A teacher may begin using Think-Aloud by describing a problem or activity for the students to do and then sitting near them with a recording device. Teachers explain the task at hand and the Think-Aloud method of narration/description. A demonstration helps students understand what they are being asked to do. Next, students confirm they understand the instructions and the teacher starts the recording device. As students progress through the activity, if they are too silent, the teacher should prompt with questions such as: "What are you doing now?" "What are you thinking?" and "What do you think will happen next?"

We model this approach when working with young students in math education. Our strategy developed as an adaptation of the work of Fosnot and Dolk (2001). As in the real-life scenarios in the Phase I activity above, we pose questions for students to solve. Then we ask them to explain their reasoning with queries such as "How did you know that?" Seeking to know whether students can comprehend multiple solutions we also ask, "Can you think of another way to explain it?"

Use Real-Life Game Math Concepts to Teach Positive and Negative Relationships: Arrow Language

Group Instructions: Next, each of the groups of students is given a set of teddy bears, bus, bus stop flag and stand, and map (Fosnot & Dolk,

2001). They create their own story about going on trips and the bears getting on and off the bus. They have to model and demonstrate their stories, including the number reasoning, to the teacher and one another explaining their reasoning.

As the students develop their arrow language to express the real-life scenario and their solution in numbers and arrows (see Figures 4, 5 and 6), they then work to construct their own number lines. The emphasis, however, is on the ability to create an Open Number Line. That is, the student may start the number line wherever they want (see Figure 4). This number line becomes their self-developed tool for solving myriads of problems going forward.

Self-Created Reasoning Tool: Integrates Addition, Subtraction, Positive, Negative and Part-Whole: Open Number Line

The foundation which is built with this use of the Rekenrek is illustrated in Figure 7 and photographed in Figure 8. A specific example of this foundation is evident in the experience of creating alternate solution combinations for different numbers with the Part-Whole approach using the Rekenrek. For instance, as seen in Figure 7, students could identify 7 as being 5 and 2, while 8 is 5 plus 3, thus demonstrating adding more to the unit of 5. The students internalize understanding the Whole as Parts, and the Parts as contributing to the sum of the Whole.

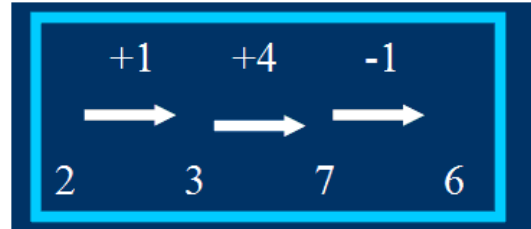


Figure 5. Using Arrow Language in Problem Solving

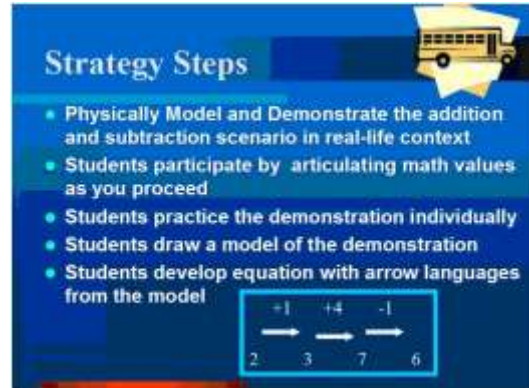


Figure 6. Strategy Steps Used by Students

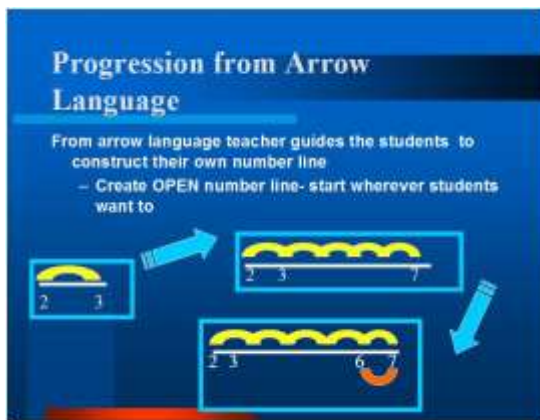


Figure 4. Representing Progression with Arrow Language on an Open Number Line

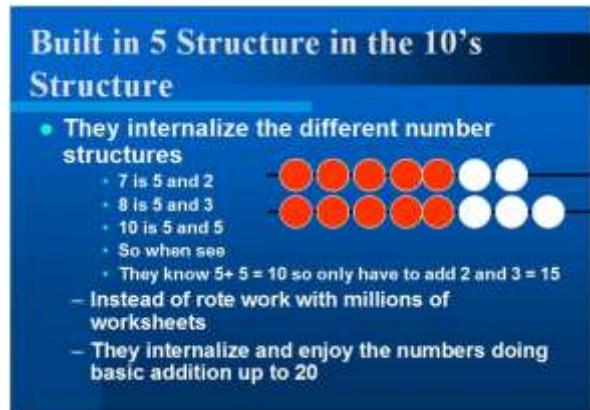


Figure 7. Internalizing concepts of 5s and 10s with the Rekenrek

Alternatively, another activity is determining the possible equations to sum a specific number: 7 is not just 5 plus 2, but instead 3 plus 4, 1 plus 6, etc. Students internalize the relationship of the numbers and are not referring to the rote number combinations learned by drill. At this young age, they now intuit the possible combinations. As discussed prior, this substantial outcome is based on the use of several different regions of the brain and sensory senses. Together this multisensory experience has laid a foundation which is essential for students to succeed in Algebraic thinking and advanced mathematics work. (See Figure 9.)



Figure 8. A Student Using a Rekenrek to Solve Math Part-Whole Problems

Double Digit Computation Internalized: Rekenrek

The second Rekenrek activity (Phase 5 of these Part-Whole activities) assists students in understanding, practicing and ultimately internalizing the concept of doubling in real-life context (See Figure 9). This is accomplished by using the Rekenrek with the teacher and proceeding through a specific sequence of activities. Once that is completed, students begin to work independently and in small groups to solve the same problems and new problems, and create problems of their own to solve.

Figure 9. Two Levels of Addition with the Double-Decker Bus Story

 A presentation slide with a blue background. The title is "Next Phase of Modeling & Demonstration" in white text. To the right is a small image of a red double-decker bus. Below the title is a bulleted list of points in white text.

- Model two levels of addition in real-life context of the double-decker bus
 - The driver asks passengers to move up to top level because bus is crowded or good things to see!
- Students participate as before
- Teachers have to let them express it in the students' OWN thinking, not our preconceived number line or method.
- The students have to develop their own strategies and number sense through the activities

Materials needed: Rekenrek, paper and crayons or markers

Overview of lesson: Introduction of the Rekenrek, student discovery of manipulative structure and reason some of the relationships and characteristics of numbers, including doubling, up to base 10.

Teacher's instructions: As instruction moves to specific content learning, teachers first illustrate and then have students explain the relationships and reasoning. The students may individually demonstrate their learning, and then work independently or collaboratively.

Cycle of activity: Model with Rekenrek, explain, document on paper, explain with Open Number Line notation.

Teacher Script for Doubling

The following script illustrates how a teacher may introduce the doubling principle using the Rekenrek:

Teacher demonstrates using the manipulative to “see” doubling relationship 1+1, 2+2, etc.

- “Can you copy my actions? What do you see? Why is it this way? What does it mean?”

Teacher continues sequencing to doubling of 10

- “Can you copy my actions? What are we doing? How do you see it? What does it mean?”

Once students understand this completely, extra beads are added to the Rekenrek.

NOTE: Teachers should only add individual beads up to the base of 10.

Math Problem Solvers

At the pinnacle or culmination of the original model of instruction (Appendix 2) is the phase of the Math Problem Solver. In addition to the students having internalized the number line and problem solving approach, they are also able to adapt it. By the end of this unit of activities, they are able to apply it to different problems and contexts. These young students have become dynamic problem solvers: they created their own number line, and they know implicitly how to use it and the method as a model to solve other problems they encounter.

Discussion

This collaborative action research among professors and teacher researchers has resulted in the above instructional sequence and strategies being developed and explored in P-4 classrooms. Therefore, the developed instructional model itself, as documented in the Findings section and illustrated in Appendix 2, is a comprehensive, macro-level outcome and result of this study.

This section discusses not only several observations and insights about this model, but also themes which emerged in the analysis of the data. As the professors and teacher researchers examined the entire body of data: classroom observations, collaborative dialogue, classroom instruction, and related video clips and photographs, student artifacts and teacher researcher reflections and presentations.

Math Problem Solvers

As stated, based on the process followed included observed classroom needs, math pedagogical instruction and action research intervention, the goal or pinnacle of the math instructional sequence developed in this project is the cultivation of young math problem solvers. Other recent research studies confirm the same powerful findings regarding problem solving and

math learning with older students. Indeed, in a recent study investigating Part-Whole relationships by Hackenberg and Tillema (2009), the schema used in problem solving (Von Glaserfeld, 1991) was a prime means for framing the strategies and efforts among students developing understanding of multiplication and fraction relationships. The constructivist activities used during problem solving not only built greater ability to solve other problems in the future (generalize findings, have additional strategies, etc), but also provided vital first hand experience for internalizing math understanding.

Returning to the study at hand in this article, several examples could be provided, but we will offer one as representative here. Based on their constructivist math learning experience using real life case scenarios and play with the teddy bears, and bus, with Think Aloud technique and number line construction, in the future, students have a new strategy for problem solving. Based on the classroom observations, it is evident that once the students discovered how they could create their own number line to solve such problems, they used it as their strategy of choice in future situations.

This problem-solving behavior is starkly different from how to students participate and respond when they learn by rote practice and repetition. Moreover, it confirms the literature and demonstrates that constructivist experience is more powerful for student understanding than rote memorization (Papert, 1980; Von Glaserfeld, 1991). In the constructivist classrooms and cases studied herein, more complex problem solving situations create confusion and lack of alternatives; the students have no schema upon which to build to develop solutions. The benefit of the real life case scenarios, coupled with problem solving, think aloud and Open Number Line, is that even at this early stage in the sequence of math learning and reasoning development, young learners are internalizing new problem solving solutions which can be generalized to other situations. The birth of math problem solvers is underway among young learners.

Two Different Types of Learning

Related to the development of math problem solvers is a very specific pattern which was also identified during the data analysis. The young students developed an ability to learn number sense and math concepts in both horizontal and vertical learning; thereby a greater breadth of learning styles was addressed successfully. This pattern became evident when the students were observed using the manipulatives independently, with the teacher and in pairs. Not only were they engaged in horizontal, but also vertical learning.

Examination of the literature confirms that there are two kinds of learning: horizontal and vertical (Stroup & Petrosino, 2003). Using the manipulatives to reenact the story helps students experience the number relationships and engage in using the manipulatives in both horizontal and vertical learning (Stroup & Petrosino, 2003). Moreover, although most students have a propensity towards one type of learning or the other, traditional methods only use the horizontal type of learning strategy (seen in $2 + 4 = 6$). Therefore, what was identified in the research study was that the manipulative, methods and strategies afforded opportunities for experience and learning math concepts and number sense in both horizontal and vertical planes. This dynamic results in a greater equity in meeting multiple learning styles among young children and the potential for a much larger percentage of students being able to comprehend this learning at young ages (NCTM, 2009).

Synergies

Examining the bigger picture of this research project, the data displays and analyses revealed several significant synergies and benefits from using these instructional methods, strategies and sequence with young children. These specific benefits regarding math teaching and learning include

The instructional methods and sequence provided

- A means for students to narrate their problem solving process (Papert, 1980),

- The establishment of an explicit working “space” for learners’ original thinking,
- A vital context for conversation with their teacher about math concepts and learning (van Someren, Barnard, & Sandberg, 1994),
- The involvement of different parts of the students’ brain by developing verbal descriptions of their thoughts and actions.
- More complex problem solving opportunities which the literature indicates includes more parts of the brain (Jensen, 2005; NCTM, 2009; Papert, 1980).
- Greater opportunities for internalization of number sense because multiple senses and parts of the brain are involved.

The individual instructional activities which the teacher researchers used with their students had good results. However, the most profound results were due to the synergistic impact of the instructional methods, manipulatives and problem solving in a coordinated sequence of discovery and learning listed above.

Specific Math Education Strategies

This research project also revealed how educators’ math pedagogical understanding of student learning undergirded instructional decision making and action research process. In the approach to learning used in this study of Part-Whole concepts, there is a rapid progression to the real development of problem-solving strategies: not merely grasping these strategies, but articulating them, documenting them and owning them.

How does this progress? What do the teachers see as common stages and math understanding development with this approach? Our analysis identified the following student learning needs and math learning sequence of development within the larger series of activities and learning (Appendix 2).

1. Students need to participate by articulating math values as the teacher proceeds through real-life examples (including Think Aloud strategies).

2. Students practice the demonstration individually and independently.
3. Students draw a model of the demonstration.
4. Students develop their Open Number Line.
5. Students develop understanding of Doubling and Part-Whole with the Rekenrek.

This sequence reveals the relationship between individual learning, demonstration/modeling, student generated problem solving methods, and internalization of math concepts. More often traditional math learning, although understood as connected by the teachers, is understood and remains compartmentalized in mental silos of concepts among young students. These strategies created an integrative experience that coordinated, integrated and synthesized several different real life activities, hands-on manipulation of toys, and tools to explore number concepts and discover number sense. Learning happened *along the way* within the context of discovery and exploration. This research project demonstrates the power of situated learning even for young learners.

More Benefits of the Rekenrek

While many benefits of the Rekenrek had been identified in our literature review, analysis of the study's data revealed several additional benefits. It became apparent that the Rekenrek leveraged several advantages and benefits related to the brain development of young students. For example, prior to first grade we had seen, and literature confirms, that children have difficulty looking with the right eye and moving the left hand, and vice versa (Lembke & Foegen, 2009). The very structure and function of the two-color Rekenrek (with five beads of each color) can help develop and master this skill.

How important is this finding and benefit? The right eye- left hand and left eye-right hand communication is so important that it is a critical "marker" developmental skill for school readiness screening for first grade placement which customarily identifies shortcomings in this area. When these skills are missing or underdeveloped, it is an indicator of lack of readiness or special education needs (Lembke & Foegen, 2009). If preschools used Rekenreks appropriately in their

math learning programs, more of their students would develop these skills which are necessary for school placement.

Another unique benefit of the Rekenrek design and use for young children is that 10 is a very large and difficult number for them. When young children attempt to do calculations with units of ten, they will usually resort to counting by ones. However, by exploring and solving real-life scenarios and word problems with the aid of the Rekenrek, in our study the same students quickly started working in units of 5. As confirmed by Andrews and Liesen (2006) and Powell, Fuchs and Fuchs (2013), this change in strategy reveals an increased level of what has been internalized in number sense. This internalization is a major leap in developing a real number sense and facilitating arithmetic skills. The following quote confirms the critical importance of this finding.

This tool [the Rekenrek] was designed to support the natural ways that children develop mathematical understandings and to encourage them to use strategies like doubles or near doubles, and "thinking 10" in place of counting from one or counting on to solve addition and subtraction problems. Along the way, children develop a better sense of number relationships that form the basis for efficient calculation and allow for quick recall of math facts. (Andrews & Liesen, p. 5)

Benefits of the Methods and Approaches

Finally, in our analysis we consistently identified several benefits of incorporating the entire math learning instructional strategies, problem solving, and sequence as described with young children. The most salient among these benefits in this context and a brief explanation of each include,

- *It encourages organizing thoughts about number sense and problem solving:*
 - Before young students can verbally express number sense and problem solving, they need to organize it in their minds.

- *It cultivates listening skills:*
 - During teacher and student exchanges, the other students learn to listen, and follow each other's thinking.
- *It provides alternate explanations:*
 - Their peer's wording is closer to their own thinking; therefore, it is easier for students to comprehend the strategies when they hear their classmates' explanations.
- *It develops cooperative learning basics and skills:*
 - The experience builds essential skills, which are the basis for cooperative learning.

While, each of these learner benefits is consistent with content standards across young grades (NCTM, 2006), another valuable set of benefits exist. The benefits of this field tested approach also include

- *It cultivates confidence in math learning:*
 - As the students solve problems successfully, their self-efficacy and confidence increase (Hidi & Harackiewicz, 2000; Wang, 2012).
 - This increased confidence results in eagerness to continue to solve math problems.
 - Based on the research of Blickenstaff (2005) and Wang (2013), this confidence could lead to continued choices of STEM courses and math as a college major.
 - Wang (2013) concluded that introducing STEM courses prior to high school could further increase math major choices, while Blickenstaff had recognized the need for exposure to such courses prior to college.
- *It motivates students to engage in math learning:*
 - Many times, the young students extended their math learning beyond the class session time.
 - They were excited to participate in the math learning activities.
 - The collaboration, real-life contexts, and manipulatives created learning experiences in which students desired to develop

effective explanations for their math learning.

- These explanations encouraged students to realize there were many math solutions to discover for a given activity.
- While enjoying these structured group activities, the students transparently learned pre-Algebraic principles.

Limitations

Action research of this nature is not without its limitation. As indicated previously, this study cannot be broadly generalizable because of its small sample size, and lack of random sample. Nonetheless, the research project does provide several leads for further research and a model for collaborative action research inquiry (Stringer, 2014).

Recommendations

While this article already provides several specific and extensive recommendations for math instruction, classroom activities, think aloud and manipulative applications, additional broader trends about teacher professional learning and research emerged from the study. First, this research project provides several math instructional strategies which would benefit from continued study in varied contexts, settings, and grades. While much research has explored the impact of STEM courses with secondary education students, little has been conducted regarding young children's self-efficacy and motivation. Regarding links among motivation, self-efficacy and math study, this current research, as well as that by Hidi and Harackiewicz (2000) and Wang (2012), provide strong arguments for encouraging young students to enjoy math learning through the structured formats described.

Second, this research provides substantial additional support for the effectiveness of collaborative action research projects in the instructional development of educators (Stein, Engel, Smith, & Hughes, 2009). Such instructional development need not be iconoclastic, but may be considered in several forms including in-service,

graduate education, or informal professional collaborations. Third, the efficacy of mathematical teaching and learning collaborative inquiry of emergent teacher researchers could be explored in continued practice and study. There is a great need for teachers to continue to hone their instructional design and math thinking skills. By engaging in collaborative professional learning, which mirrors the classroom activities they facilitate, the teachers better understand student collaborative learning needs.

Conclusion

Based on our understanding as educators, when teachers better understand the development of students' thinking and outcomes of practice, they are more ready and able to learn and implement new instructional practices (Schon, 1987). This article delineated how to use the Rekenrek manipulative in a manner which can afford a rapid development of number sense and a strong foundation for Algebraic thinking among young students. Moreover, it provided detailed examples of how to use real-life scenarios and the Think-Aloud method to accomplish these outcomes.

This approach may be considered a more advanced application of constructivism than many math teachers have previously seen. In our experience, effective constructivist instructional activities do not have to be difficult to design or implement. Indeed, the approach described herein uses simple scenarios and explanations, dialogue among teacher and student, and individual and small group exploration of knowledge building. The result is a teacher-accessible multi-sensory approach to teaching and learning which yields the best of Constructivism. From cultivating student engagement, to facilitating problem solving, collaborative learning, self-directed learning experiences, and positive experiences in math learning (Papert, 1980; Von Glaserfeld, 1991), advancing the understanding of content knowledge on an internalized level, but this Part-Whole approach to math learning illustrates that young children gain many benefits from well-designed Constructivist learning experiences.

These strategies offer opportunities to create new futures for the youngsters of tomorrow. Instead of continuing the pattern of large numbers of students and adults struggling with math, preschoolers who learn Part-Whole concepts in this manner will have confidence and motivation for math learning, as well as a strong foundation of number sense and math concepts. Both these affective and cognitive results can yield much needed advancement for STEM learning and career choices.

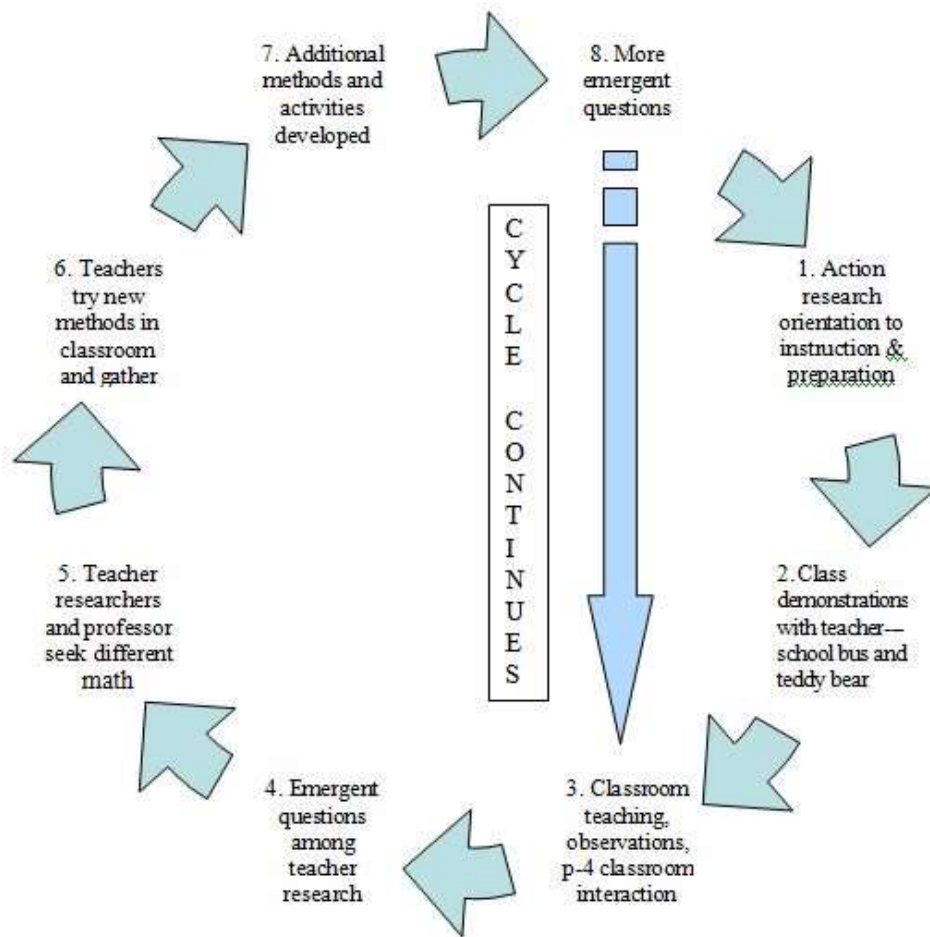
References

- Andrews, A., & Liesen, D.C. (2006). *Reasoning with the Rekenrek*. Holland: Jegro.
- Blickenstaff, J. C. (2005). Women and science careers: Leaky pipeline or gender filter. *Gender and Education, 17*(4), 369–386.
- Charmaz, K. (2014). *Constructing grounded theory*. (2nd ed.). Thousand Oaks, CA: Sage.
- Crane, S. W. (2013). Measuring early numeracy of kindergarten students in a group setting. *Dissertation Abstracts International Section, A, 73*,
- Creswell, J. (2014). *Research design* (4th ed.). Thousand Oaks, CA: Sage.
- Fosnot, C. T. & Dolk, M. (2001). *Young mathematicians at work*. Portsmouth, NH: Heinemann.
- Glaser, B. G., & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research*. Chicago, IL: Aldine.
- Hackenberg, A., & Tillema, E. (2009). Students' whole number multiplicative concepts. *Journal of Mathematical Behavior 28*, 1-18.
- Hidi, S., & Harackiewicz, J. (2000). Motivating the academically unmotivated. *Review of Educational Research, 70*, 151–179. doi:10.2307/1170660
- Hinchey, P. H. (2008). *Action research primer*. Thousand Oaks, CA: Sage Publications.
- Izzo, A. (2006). *Research and reflection*. Charlotte, NC: Information Age.
- Jensen, E. (2005). *Teaching with the brain in mind* (2nd ed.). Alexandria, VA: ASCD.
- Kamii, C. & Dominick, A. (1998). The harmful effects of algorithms in grades 1-4. In L. Morrow and M. Kennerly (Eds.). *The*

- teaching and learning of algorithms in school mathematics* (pp 130-140). Reston VA: NCTM.
- Lembke, E., & Foegen, A. (2009). Identifying early numeracy in kindergarten and first grade students. *Learning Disabilities & Research Practice* 24(1), 12-20.
- Lyublinskaya, I., & Kerekes, J. (2008). *Integrating mathematics, Science and Technology in the elementary classroom*. Oceanside, NY: Whittier Publications, Inc.
- NCTM. (2009). *Equity means all*. Retrieved from <http://www.nctm.org/profdev/content.aspx?id=15589>
- NCTM. (2006). *Focal points for the mathematics curriculum*. Reston, VA: NCTM.
- Nevarez LaTorre, A. (2010). *The power of learning from inquiry*. Charlotte, NC: Information Age.
- Papert, S. (1980). *Mindstorms*. New York, NY: Basic Books.
- Powell, S. R., Fuchs, L. S., & Fuchs, D. (2013). Reaching the mountaintop. *Learning Disabilities Research & Practice (Wiley-Blackwell)*, 28(1), 38-48. doi:10.1111/ldrp.12001
- Schon, D. (1987). *Educating the reflective practitioner*. San Francisco, CA: Jossey-Bass.
- Stein, M. K., Engel, R. A., Smith, M. S., & Hughes, E. K. (2009). Orchestrating productive mathematical discussions. *Mathematical Teaching and Learning* 10, 313-340.
- Stringer, E. (2014). *Action research*. (4th). Thousand Oaks, CA: Sage.
- Stroup, W., & Petrosino, A. (2003). An analysis of horizontal and vertical device design for school-related teaching and learning. *Education, Communication and Information* 3(3), 327-345.
- Tournaki, N., Bae, Y. S., & Kerekes, J. (2008). Rekenrek. *Learning Disabilities: A Contemporary Journal* 6 (2), 41-59.
- Treffers, A. (1997). Rekenonderwijs naar menselijke maat. *Willem Bartjens* 17 (1), 5-8.
- Treffers, A. (1998). *Tussendoelen Annex Leerlijnen. Hele getallen onderbouw Basis-school* (p.45). Utrecht: Freudenthal Instituute, SLO en CED.
- van Someren, M. W., Barnard, Y. F., & Sandberg, J. A. (1994). *The think aloud method: A practical guide to modelling cognitive processes*. London: Academic Press.
- von Glaserfeld, E. (1991). *Radical constructivism in mathematics education*. Dordrecht, Holland: Kluwer Publishing.
- Walcott, C., Mohr, D., & Kastberg, S. (2009). Making sense of shape: An analysis of children's written responses. *Journal of Mathematical Behavior* 28, 30-40.
- Wang, M. (2012). Educational and career interests in math: A longitudinal examination of the links between classroom environment, motivational beliefs, and interests. *Developmental Psychology*, 48(6), 1643-1657. doi:10.1037/a0027247
- Wang, X. (2013, Oct.). Why students choose stem majors: Motivation, high school learning, and postsecondary context of support. *American Educational Research Journal* 50(5), 1081-1121. doi:10.3102/0002831213488622

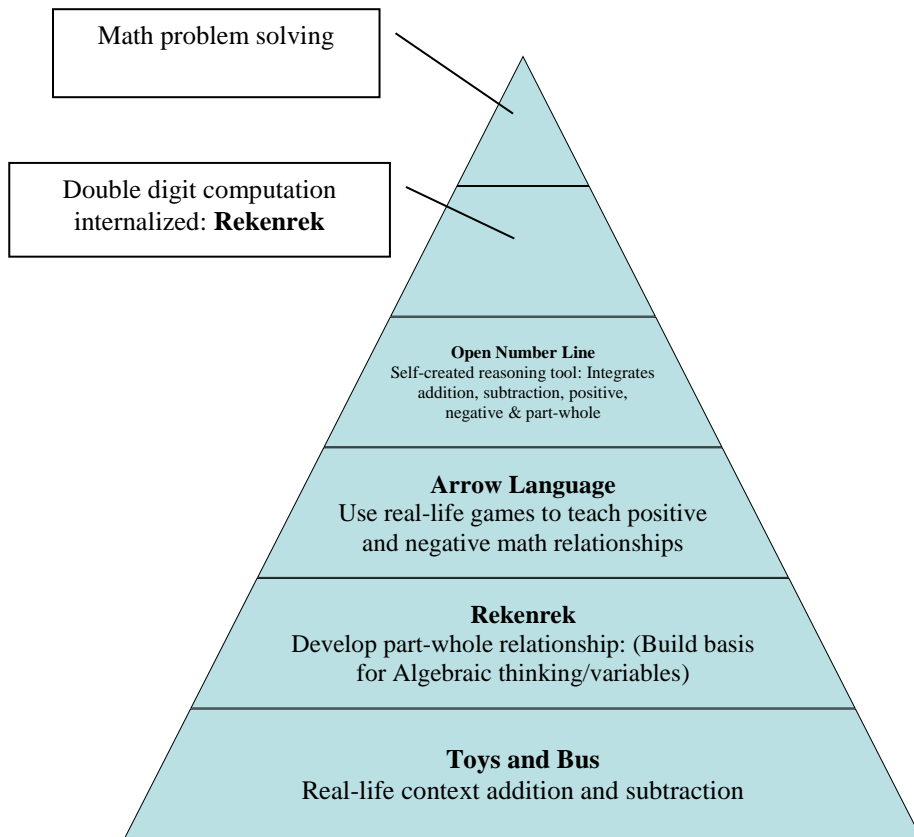
Appendix 1. Magnified View of Figure 1: Action Research Cycle for Teachers' Professional Learning

Kerekes, J., & King, K. P. (2015). Creating dynamic problem solvers while learning part whole concepts: Young children using manipulatives for mathematics learning, *Mathitudes 1(1)*, Page 14 of 17.



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Appendix 2. Sequence of Young Students' Developing Understanding of Part-Whole Relationships Using Manipulatives and Real-Life Activities



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